University of Alberta

Evaluation of the Real Options Approach to Agribusiness Valuation: A Pork Investment Case Study.

by

Armstrong Duku-Kaakyire

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements of the degree of Master of Science

in

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Abstract

The application of financial option theory to the valuation of capital projects is termed the real options approach. The purpose of this study is to determine the applicability of the real options approach to valuing investments in agriculture. Specifically, the study developed an NPV and real options models to evaluate a farrow-to-finish pork investment.

Although the NPV analysis rejected the hog project evaluated, the value of the project improved when the managerial options were considered. Particularly, the option to wait was highly valuable for this investment. However, if the expected average price of finished hog increased by between 23%-29%, it would be economically sensible for management to invest immediately. The sensitivity analyses indicate that the NPV and the real option results are sensitive to the discount rate and the volatility respectively. The results of the option analysis closely matched the actual decision of the management of the case studied.

DEDICATION

This thesis is dedicated to my dear mother, Miss Cecilia J. Addai. I am who I am today because of you. May the Almighty God continue to bless you with His abundant wisdom and knowledge.

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CHAPTER 1: BACKGROUND

Effective capital budgeting is important in any major investment decisions. In the process of capital budgeting, managers make decisions that commit the firm's resources across business lines and over time. The field of capital budgeting has undergone a significant change since it was recognized that the option inherent in strategic capital allocation could be valued by building on the work of Black and Scholes (1973), Merton (1973) and others, and applying this to capital projects. Hitherto, the main approach to capital budgeting decision by corporate managers was the use of the traditional capital budgeting techniques, such as net present value (NPV). This conventional way of allocating resources often underestimates project value by failing to adequately address project valuation of growth opportunities or strategic alternatives arising from investments in large-scale agri-business or commercial investment in high-risk projects (Trigeorgis, 1991). For instance, the flexibility to optimally time project initiation is often ignored by NPV, which implicitly assumes that immediate acceptance and rejection decision needs to be made. However, the ability to wait and see could be valuable to managers, especially when the project value changes stochastically.

The limitations of the traditional capital budgeting techniques have resulted in the questioning of the validity of valuations made using such techniques. Some researchers are of the view that these limitations of traditional capital budgeting could render the conclusions of project valuation as somewhat suspect (Trigeorgis, 1991), especially projects with high risks and uncertainties. Others researchers are of the view that traditional capital budgeting is an appropriate model but it is often misapplied (Hodder, 1986; Aggarwal, 1993). Nonetheless, the traditional capital budgeting methods may not fully capture the strategic value of an investment. In an attempt to model uncertainty and complexity in the traditional capital budgeting settings, and to further account for flexibility, managers use other approaches such as simulation and decision tree analysis (Trigeorgis, 1999). Although these approaches are helpful in improving management understanding of the investment structure, they still fall short of offering a manageable consistent solution.

The above shortcomings of the traditional approach increased the need for a richer and more efficient capital budgeting decision making framework that directly translates into increased corporate effectiveness, profitability, and long-term survival in today's globally competitive market place. The application of financial option theory to valuing capital projects is termed the real options approach. Real options allow managers to add value to their firm, by acting to amplify good fortune or to mitigate loss (Brealey and Meyers, 1991). The real options literature and methodology provides a way to value managerial flexibility, such as, option to wait, option to abandon and other strategic managerial options that may be present in an investment opportunity. These strategic managerial options often have considerable value especially in the context of the significant amount of risks and uncertainties associated with investments in agriculture. When real options are present, the traditional methodology may fail to provide an adequate decision-making framework because it does not properly value management's ability to wait, to revise the initial operating strategy if future events turn out to be different from originally predicted, or to account for future investments or disinvestments (Lander and Pinches, 1998).

Real options have received increasing attention over the last decade, both from academics as well as corporate practitioners. Early application of the real option approach focused mostly on the natural resource industries, particularly in oil extraction (e.g., Tourinho, 1979; Brennan and Schwartz, 1985; Smith, 1997; Tufano, 1998) and the pharmaceutical industries (Nichols, 1994; Perlitz et al., 1999). This is due to the fact that these industries are faced with highly uncertain project costs and markets and therefore see the real option approach as a more appropriate way of valuing uncertain investments. Despite the attention received so far, the adoption of the real options approach by corporate managers and practitioners has been slow (Lander and Pinches, 1998). One reason to this relates to the fact that most of the real options literature uses difficult mathematical solutions (e.g., Ito calculus, partial differential equations, etc.) that are not easy to understand by corporate managers and practitioners. Copeland and Antikarov (2001) contend that the real options approach needs to be simplified in order for management to understand and appreciate the approach. This study uses lattice and algebraic solutions that are relatively easier to understand and implement.

Investments in agriculture entail significant amount of risk and uncertainties, which makes strategic managerial decision-making important. Particularly, the establishment of a farrow-to-finish hog operation is characterised by a relatively large sunk cost as well as a significant amount of risk and uncertainties both in production and marketing. The sunk cost in farrow-to-finish hog production results from the cost of constructing the barn, which may be partly or wholly irretrievable. Due to the irretrievable nature of such investments, it is important that greater focus is placed on the investment valuation. According to Bodie and Merton (2001), delaying an irreversible expenditure involves the issue of managerial flexibility to alter the course of the investment in the future. This managerial flexibility is important in structuring the investment analysis, as it is often a source of additional value in the investment decisions.

Investment decisions in projects, such as described above, are complicated and include not only tangible future cash flows but also intangible factors such as information gained through research, possible follow-on projects and other managerial flexibilities (Nichols, 1994). Thus, the application of real options valuation to investments in agriculture is promising. However, investments in the agricultural sector have seen limited application of the real options approach (Boehlje and Lins, 1998). Price and Wetzstein (1999), Bailey and Sporleder (2000), Morck et al. (1989), Maung and Foster (2002) are some of the few examples of studies that have used the real options approach to evaluate investments in agriculture and resource sectors. However, none of the above studies extend the real options approach to assess the values of the range of managerial flexibility that may be available to managers of agricultural firms.

1.1 Objective of Study

The main objective of this study is to evaluate the applicability of the real options approach to the valuation of an investment in a farrow-to-finish pork project. Specifically, the study evaluates a 2600 farrow-to-finish pork investment using both the traditional budgeting technique (static NPV) and the real options approach. The following assessments are made:

1. The value of the project from both methodologies is compared to assess the impact of strategic managerial decision making on the project.

2. The result of each methodology is compared to the actual decision made by the investors of the hog project.
3. Based on 2, the study concludes on whether the real option approach would be a useful[1] capital budgeting technique for investors in hog production and other similar investments in agriculture.

The study makes the following contributions related to the above objectives. First, the data used for the analysis are from a business that has actually invested in a farrow to finish hog operation. This makes it possible for the analysis to be compared to the actual decision taken by the business. Second, while many different options have been examined and different techniques developed for valuation, the flexibility and complexity inherent in most agricultural projects have not been addressed. This paper demonstrates that recognizing the options inherent in a hog investment opportunity and quantifying the values of such options can have a significant impact on the value of the investment and hence capital budgeting decision-making. The important point demonstrated from the implementation of the real option methodology to the hog investment project is that options can be recognized as inherent in such projects and could add value. Coming up with the appropriate value for the options may be sufficient to properly assess a project's value. Third, most real options applications use complex mathematical tools that are difficult to understand. This is one of the contributing factors to the limited application of the real options approach since corporate managers and practitioners find it difficult to understand. However, this paper shows that the approach can be simplified and that most of the variables in the option model can be determined from the already familiar NPV without flexibility approach. In addition, the paper explains in detail how each of the real options is evaluated. Interested managers or investors can work through real options problems by following the steps provided by this study and understand the results of the analysis. Finally, the information provided by this report should be of significance not only to existing and would be investors in the hog industry, but also to investors in other agricultural industries, particularly those that face similar risk characteristics as the hog industry.

1.2 Methodology

To evaluate the possibility of using real option theory to analyse investments in agriculture, a comprehensive literature review is carried out in the areas of traditional capital budgeting techniques, financial and real options theories, as well as a brief overview of the Canadian hog industry. The literature review section is based mainly on secondary sources such as published books, journal articles, reports and the internet.

To quantitatively evaluate the real options inherent in the hog project, the analysis makes use of the multiplicative binomial model developed by Cox et al. (1979). Although other approaches to valuing real options are available, they adopt more complicated mathematical solutions that are usually difficult to understand by practitioners. The binomial model presented in this thesis tries to give a simplified approach to the numerical valuation process.

---

[1] Usefulness is defined in terms of whether the real options approach would be a potential capital budgeting tool that could be applied to the valuation investments in hog production and other similar investment in agriculture.

1.3 Structure of the Study

The thesis is structured in two main parts. The first part consists of a literature review, which is focused on the fundamentals of the traditional capital budgeting and real option theories. The second part consists of evaluating a 2600 farrow-to-finish pork investment. The detailed structure of the thesis is as follows:

The literature review part starts with a discussion of the underlying theory and methods of traditional capital budgeting techniques along with their limitations in chapter two. The main focus of discussion is on the NPV since it forms the basis of determining some of the variables required for real option analysis. Some of the extensions made to NPV analysis such as simulation and decision tree analyses are discussed briefly. The fundamentals of real options theory and valuation techniques are discussed in chapter three. The financial and real options analogy is introduced in this chapter along with the different types of real options that could provide managers with operating flexibility. This is followed next with a discussion of the real options valuation techniques. Finally the shortcomings and critique of the real options approach is presented. The literature review part of this study explains the basic concepts of the methodologies used and how they have been applied, particularly the real options approach.

The second part consists of a case study of a farrow-to-finish pork operation in Western Canada. Brief overviews of the pork case study and the Canadian hog industry as well as the main sources of risk faced by pork producers are discussed in chapter four. This chapter also discusses the possible real options in the pork project and how these options are evaluated. Chapter five presents the valuation and results of the project analysis. In chapter six, a sensitivity analysis of the impact of the key variables in the models is conducted. Conclusions and the direction of further studies are presented in chapter seven.

CHAPTER 2: TRADITIONAL CAPITAL BUDGETING MODELS

The traditional capital budgeting analysis is a very popular financial technique (Arthur et al., 1997; Graham and Harvey, 2001), integrating many of the basic concepts of finance theory. Various traditional methodologies have been used to ascertain the financial viability or otherwise of an investment project. The most common traditional capital budgeting model used by corporate managers is the net present value (NPV) approach (Trigeorgis, 1999; Copeland and Antikarov, 2001). In an attempt to deal with uncertainty and complexity, and to account for the possibility of later decisions, managers adopt other approaches such as simulation and decision tree analysis (Trigeorgis, 1999). These three different capital budgeting techniques mentioned above are discussed along with their shortcomings in this chapter. More emphasis is however laid on the NPV approach and how it could be useful for the real options analysis. Other traditional approaches like the internal rate of return (IRR), the payback period (PBP) and the accounting rate of return (ARR) are briefly discussed as well.

2.1 Net Present Value (NPV) Approach

The NPV approach is one of the most common traditional techniques used in capital budgeting, particularly for large investments (Graham and Harvey, 2001; Copeland and Antikarov, 2001). The NPV methodology measures the difference between the present values (PV) of the expected cash inflows and outflows of an investment. It discounts the expected net cash flows from the proposed project with an appropriate discount rate. There are two main approaches of determining the NPV of an investment (Trigeorgis, 1999; Copeland and Antikarov, 2001). The first is the risk adjusted discount rate approach and the second is the certainty equivalent approach[2].

The risk adjusted discount rate approach discounts the expected cash flows of an investment with a discount rate, which is adjusted to include a risk premium appropriate for the project (Copeland and Antikarov, 2001). This approach accounts, through the discount rate, for both the time value of money and uncertainty (risk aversion) (Trigeorgis, 1999). Thus, the risk-adjusted discount rate is the sum of the risk free rate used to discount for the time value of money and a risk premium to compensate investors for the risk associated with the project. With a constant discount rate[3], the risk adjusted discount rate approach is algebraically expressed as (Trigeorgis, 1999, Copeland and Antikarov, 2001):

$$\text{NPV} = \sum_{t=1}^{T} \frac{E(c_t)}{(1+k)^t} - I = V_0 - I, \qquad (2.1)$$

where

[2] For a detailed discussion of both approaches see Trigeorgis (1999), Copeland and Antikarov (2001) and other corporate finance textbooks.

[3] It is possible to have a variable discount rate, but this is not considered in this study. Readers are referred to Trigeorgis (1999) for an example of NPV calculation with variable discount rates.

$E(c_t)$ = the expected free cash flows in year t
k = the risk adjusted discount rate
I = the PV of investment outlay at time 0
V_0 = the gross present value of the project.

The certainty equivalent approach, on the other hand, discounts the certainty equivalent cash flows (i.e., the certain cash flows that has the same present value as the uncertain expected cash flows discounted with a risk adjusted discount rate) with a risk free discount rate. This approach accounts for the time value of money through the risk free discount rate and risk aversion through the certainty equivalent cash flows (Trigeorgis, 1999). The certainty equivalent approach is expressed as (Trigeorgis, 1999; Copeland and Antikarov, 2001):

$$NPV = \sum_{t=1}^{T} \frac{\hat{c}_t}{(1+r)^t} - I = V_0 - I \qquad (2.2)$$

where

$\hat{c}_t = E(c_t)\left(\frac{1+r}{1+k}\right)^t$ = the certainty equivalent free cash flow in year t

r = the risk-free discount rate

As will be shown later, the certainty equivalent approach reaches the same results as the risk adjusted discount rate approach for a particular investment. The rule with the NPV approach is that, an investment is acceptable if it has a positive NPV.

The NPV approach is the main quantitative capital budgeting technique used for major investment appraisals (Graham and Harvey, 2001; Copeland and Antikarov, 2001). According to Trigeorgis (1996), the NPV approach is the only currently available valuation measure consistent with a firm's objective of maximizing its shareholder's wealth if there is no managerial flexibility. The NPV model is considered to be a superior valuation model compared to other traditional valuation methods like IRR, ARR and PBP (Brigham and Gapenski, 1997; Trigeorgis, 1999). Other reasons that have facilitated the use of the NPV approach is its simplicity and the fact that it takes into consideration the time value of money (Herbst, 1982; Brigham and Gapenski, 1997).

2.1.1 Limitations

The NPV approach has been criticized for a number of reasons. One of the standard criticisms of the approach relates to the determination of the relevant risk adjusted discount rate. Determining the appropriate risk adjusted discount rate is crucial in cash flow analysis (Butler and Schachter, 1989), because it adjusts the project analysis for risk. This requires sound judgement guided by knowledge of economic indicators and market attitudes. In addition, the NPV approach does not shed light on crucial decisions that may affect a project's value. In other words, it does not deal with management's ability to time its decisions to take maximum advantage of the riskiness in the cash flows (Trigeorgis, 1999, Copeland and Antikarov, 2001). The NPV rule implicitly assumes that the investment falls into one of two categories; that either the investment is reversible or irreversible (Dixit and Pindyck, 1994; 1995). The reversible category assumes that expenditures can be fully recovered if market conditions turn out to be worse than

expected, while the irreversible category assumes that the investment is a now or never proposition. In some cases however, investments are irreversible and could be delayed but the NPV approach is not able to properly capture the value of investments in this category (Dixit and Pindyck, 1995). As a result, NPV analysis may underestimate investment opportunities because it ignores management flexibility to alter initial decisions, as new information becomes available (Trigeorgis, 1991; 1999). The implications are that, not only will managers select investments that turn out to be unprofitable, but may fail to undertake very risky, but strategically vital ones (Sharp, 1991).

Another limitation of the static NPV is its failure to properly account for the effects current investments may have on future opportunities by creating growth or by reducing the value of an existing opportunity (Baldwin, 1982). The reasoning is that in some instances it may be worthwhile to invest when NPV is negative because of the growth opportunities that could be created by the investment. On the other hand, it may not be worthwhile to invest although NPV is positive because of reduced opportunities that may be created by the investment. According to Roberts and Weitzman (1981), if an investment requires a sequential decision-making and if early investment can reveal information about the future profitability of the project, it may be worthwhile to invest even when NPV is negative. This is particularly so if the project is associated with high uncertainty. Balwin (1982), on the other hand, found that optimal sequential investment for firms, which have market power and are faced with investment irreversibility, may require a positive investment premium over NPV. Balwin (1982) contends that the premium is necessary to compensate the firm for the loss in the value of the future opportunities implicit in undertaking the investment. Other studies that have arrived at similar conclusions include Pindyck (1988), Dixit (1989), Kogut and Kulatilaka (1994) and Bell (1995).

The above limitations notwithstanding, the traditional NPV technique plays a key role in real option analysis (Copeland and Antikarov, 2001). This is because the traditional NPV approach serves as a starting point of real option analysis. As will be shown later in example 1, the gross present value of the project without flexibility has to be determined as a base case for evaluating the real options available to a project and this is done using the underlying concept of the NPV approach. Following next is a discussion of how risk premiums are determined for NPV analysis.

2.1.2 Determination of Risk Premium

Quantifying risk is essential to project analysis. Once risk is introduced, the investment problem becomes complex and in general, additional information as to individual risk attitudes must be incorporated into the analysis. Every investment project has a certain amount of risk associated with it. With the assumption that investors are generally risk averse, they demand higher returns (premium) from more uncertain investments (Megginson, 1997). According to Trigeorgis (1999), investment projects can have two main types of risks; systematic and unsystematic risks. Systematic risk arises from the correlation between the project returns and the market returns. On the other hand, unsystematic risk, also known as unique risk, is risk that is specific to the investment project. Thus, if an investor can diversify his or her investment portfolio (i.e., hold more than one investment in a portfolio at one time), unique risk can be reduced or

eliminated (Trigeorgis, 1999). However, investments in agriculture, such as farrow to finish hog projects, are often non-diversified and as a result both unique and systematic risks must be taken into consideration when analyzing such projects. The following section discusses two of the methods used to adjust the discount rate for risk in an investment.

2.1.2.1 Capital Market Line

The Capital Market Line (CML) combines assumptions regarding individuals and the market to derive the most efficient portfolios (Brigham and Gapenski, 1997). This line represents portfolios formed by combinations of risk free assets and risky portfolios (Damodaran, 1997). These portfolios have the highest return for a given level of risk: where risk is measured by the standard deviation of returns[4]. According to Brigham and Gapenski (1997), the CML provides a linear relationship between the risk and return for efficient portfolios of assets. The equation for the capital market line is given as (Copeland and Weston, 1983; Brigham and Gapenski, 1997):

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m}\sigma_p \qquad (2.3)$$

where $E(R_p)$ is the individual expected asset rate of return, R_f is the risk free rate, $E(R_m)$ is the expected market rate of returns, σ_m is the market risk and σ_p is the risk associated with the asset. If a project forms a large portion of an investor's portfolio, the CML can be used to determine the discount rate for the project (Miller, 2002). The discount rate provided by equation (2.3) consists of a risk free rate and a risk premium which is equal to the $[E(R_m) - R_f]/\sigma_m$, multiplied by the investment's standard deviation, σ_p.

The CML takes into consideration the total risk (both systematic and unsystematic risks) of a project when determining its discount rate. As a result, the CML is a more appropriate risk measure for non-diversifiable investment projects like investments in agriculture.

2.1.2.2 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM), developed by Sharpe (1964) and others, provides a method to simultaneously relate the required return of a project or security to its relevant systematic risks. In other words, the CAPM describes how the market reaches a price for risk through the formulation of a market efficient portfolio, *M*, and a risk free security, *r*, and how individual securities are priced through their covariance relationship with *M* (Levy and Sarnet, 1990). The basic idea behind the derivation of CAPM is that investors can avoid much of the unsystematic risk through diversification. As a result, the model assumes that a portfolio's expected return is based solely on its systematic risk, also referred to as beta (Brigham and Gapenski, 1997). Thus, the CAPM helps measure portfolio risk and the return an investor can expect for taking that risk.

A number of assumptions underlie the derivation of CAPM. The basic assumptions are the following (Sharpe, 1964; Trigeorgis, 1999):

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[4] Details on deriving the efficient frontier portfolio can be found in Copeland and Weston (1983), Brigham and Gapenski (1997) and most finance textbooks.

1. The objective of investors is to maximize their expected utility of wealth, i.e., investors are rational.
2. Investors diversify their portfolio efficiently based on the mean and variance of portfolio returns, i.e., they are risk averse.
3. Expectations about asset returns are homogenous, that is, identical estimates of the expected values, variances, and covariances of returns for risky assets.
4. A risk free interest rate exists and investors can borrow and lend any amount at this rate.
5. No taxes, no transaction costs, cost of bankruptcy is negligible and information is freely available to investors.
6. All assets are perfectly divisible and liquid.
7. The market is competitive (i.e. investors are price takers).

Based on the assumptions, CAPM determines the expected returns required by investors to be compensated for any level of systematic risk associated with an asset or portfolio in equilibrium as follows (Sharpe, 1964; Trigeorgis, 1999)[5]:

$$E(R_j) = R_f + \beta_j[E(R_M) - R_f] \qquad (2)$$

where $E(R_j)$ is the expected rate of returns on asset j, R_f is the risk free rate, $E(R_m)$ is the expected market rate of return, and $\beta_j \equiv Cov(R_j, R_M)/Var(R_M)$ is the asset's beta or a measure of the level of the asset's systematic risk. If an asset has no systematic risk, beta is equal to zero and the expected rate of returns for the asset would be the risk free rate.

The expected rate of returns given by CAPM provides no compensation for risk unique to the investment. Thus CAPM or similar equilibrium models are useful for investors who hold diversified portfolios but may not provide adequate risk measures for investments in agriculture or other proprietary firms where investors find it difficult to diversify.

Example 1:

The purpose of this example is to demonstrate the difference between the traditional NPV, the decision tree analysis, and later the real options techniques to capital budgeting. This is based on a simple example from Copeland and Antikarov (2001, pg 87 – 93). It is a simplified investment where the investor has to pre-commit to a project with an initial investment of $115M next year, with certainty. Once the investment is made, the project has a 50-50 chance of generating either $170M if the market responds favourably to the project or $65M when market conditions turn to be unfavourable. The risk adjusted discount rate for the investment is assumed to be 17.5% and the risk free rate is 8%. Table 2.1 summarizes the cash flow information for the project.

In this example, the investment is analyzed with the traditional NPV approach by assuming pre-commitment now to invest in year 1 or lose the opportunity. Thus, managerial flexibility is not considered and the NPV is calculated by simply discounting all the expected cash flows from the project with the appropriate discount rate.

The expected gross present value of the project, V_0, can be calculated by using the risk adjusted discount rate approach as follows:

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[5] Detailed derivation of the model can be found in Sharpe (1964), Trigeorgis (1999) and most other finance textbooks.

$$V_0 = \frac{0.5(\$170) + 0.5(\$65)}{(1+0.175)^1} = \frac{\$117.5}{1.175} = \$100$$

To demonstrate that the risk adjusted discount rate and the certainty equivalent approach gives the same results, the gross present value of the project is estimated again using the certainty equivalent approach. The certainty equivalent cash flows in the up and down states are respectively calculated as:

$$\hat{c}_u = \left(\frac{1+0.08}{1+0.175}\right)\$170 = \$156.26$$

$$\hat{c}_d = \left(\frac{1+0.08}{1+0.175}\right)\$65 = \$59.74$$

To calculate the gross present value, the certainty equivalent cash flows from the up and down states are discounted at the risk free rate as follows:

$$V_0 = \frac{0.5(\$156.26) + 0.5(59.74)}{(1+0.08)^1} = \$100$$

This is the same as the figure obtained using the risk adjusted discount rate approach. Since the initial cost of the investment is to be incurred with certainty, it is discounted with the risk free rate. Thus, the present value of the initial cost, *I*, is calculated as follows:

$$I = \frac{\$115}{(1+0.8)^1} = \$106.48$$

The NPV is thus calculated as:

$$NPV = V_0 - I = \$100 - \$106.48 = -\$6.48$$

The investment has an NPV of -$6.48 million and should therefore be rejected. As stated earlier, the NPV approach determines the starting values of some of the variables in the real options framework. As will be shown in chapter 3 when the real options approach is used to evaluate the above example, the gross project value, V_0, will be the starting value of the binomial tree. The initial cost of the project, *I*, is also used as the exercise price when the option to wait is evaluated. As a way of making up for some of the limitations of the traditional NPV, other approaches have been used. The next two sections discuss two of such approaches.

2.2 Monte Carlo Simulation

One of the extensions often made to the static (traditional) NPV analysis to account for uncertainty is the use of simulation analysis (Trigeorgis, 1999). Simulation is an analytical technique that relies on repeated random sampling from the probability distributions of the main variables underlying the cash flows of the project to arrive at risk profiles of the cash flows or NPV (Evan and Olsen, 1998). It is an analytical method meant to imitate real world decision setting by using a mathematical model to capture the important characteristics of the project as it evolves through time and encounters random events. A common type of simulation approach is called Monte Carlo. The Monte Carlo

Simulation analysis usually follows the following steps (Evans and Olson, 1998; Trigeorgis, 1999):

1. Identify all the uncertain variables in a project's cash flow set up, noting the interdependencies between different variables and any serial dependency.
2. For each uncertain variable, define the possible values with a probability distribution. The type of distribution selected is based on the nature of uncertainty surrounding that particular variable.
3. Using a mathematical model with computer software such as @Risk (Palisade Corporation 1998), a random value is then selected for each uncertain variable based on its probability distribution to calculate the net cash flows for each period and subsequently the NPV.
4. The process is repeated in a number of iterations, which results in a probability distribution, expected value and standard deviation for the project's NPV.

According to Trigeorgis (1999), the simulation approach has the advantage of handling complex and uncertain decision problems and also takes into consideration the interaction of the variables with one another and across time. However, the method has its limitations. The first limitation relates to the complexities involved in the estimation of the interdependencies and the serial dependencies. This could be a difficult and time-consuming process, which often leads management to delegate this task to experts (Trigeorgis, 1999). As a result management may not properly understand the simulation results which may consequently affect their faith in it. Secondly, the interpretation of the probability distribution of NPV given by simulation analysis is questionable, since it not clear on how to value the risk-return trade off (Evans and Olsen, 1998). Furthermore, the simulation approach cannot handle well the asymmetries in the distributions brought about by management's flexibility to alter the course of the project if, as uncertainty gets resolved over time, cash flow realizations differ from initial expectations (Trigeorgis, 1999).

2.3 Decision Tree Analysis (DTA)

All the models discussed above are based on the assumption that investment decision is now or never. However, DTA approach to capital budgeting accounts for uncertainty and later decisions made by management in an attempt to capture the value of managerial flexibility contingent on a project (Copeland and Antikarov, 2001). The approach provides an effective structure in which all feasible alternative decisions and the implications of taking those decisions are laid down and evaluated. As a result, it is possible to analyse complex sequential investment decisions when uncertainty is resolved at discrete points in time (Trigeorgis, 1999). Thus, DTA forms an accurate and balanced picture of the risks and rewards that can result from a particular choice. The general structure of a decision tree is presented in Figure 2.1.

Generally, decision trees are structured with all the possible alternatives available to management identified at the different nodes of the tree. Management is then faced with a decision of choosing the best alternative that is consistent with the maximization of the project's net present value. In Figure 2.1, the square boxes indicates decision nodes, i.e., where a decision is made and the circles indicates outcome nodes, i.e., where nature has its way and management cannot control the outcome. The branches give all the

possible options from which management can choose and the implications of choosing each option are illustrated as an additional option.

Example 2:

In an attempt to capture the value of managerial flexibility, the investment project valued in example 1 with the traditional NPV technique is re-evaluated with the decision tree approach. Suppose instead of pre-committing to undertake the investment, a mutually exclusive alternative of waiting for the end of the period before committing is considered. Figure 2.2 illustrates the various alternatives investment decisions available to the project under this alternative.

The decision tree allows management to evaluate the alternative of deferring the investment until the end of the period and decide on whether to invest \$115M depending on the state of nature. In the lower branch of the tree, the cash flow is only \$65M so it would not pay to invest \$115M, which will result in a negative NPV of \$50M. Management would choose to not invest in the project in this outcome and the net payoff would instead be \$0. The expected net payoff for the project is now calculated as follows:

$$0.5(\$170 - \$115) + 0.5(\$0) = \$27.5$$

The net present value of the decision with the right to defer is estimated by discounting the expected cash flows at the risk-adjusted discount rate as:

$$\text{NPV} = \frac{\$27.5}{(1+0.175)^1} = \$23.40$$

With the decision tree approach, the value of the project has increased to \$23.4M with the ability to defer as compared to the \$-6.48M given the NPV without flexibility. With this example, the value of the flexibility to defer with the DTA approach is the difference between the value of the project given the static NPV and the value from the decision tree analysis (i.e., \$23.4 – (-\$6.48) = \$29.88).

Although the DTA model considers the flexibility of managers, the approach is not without it limitations. The model instead of being a decision tree could become a very complex decision 'bush', particularly when the variables and the outcomes increase (Trigeorgis, 1999). The major criticism of DTA relates to how to determine the right discount rate for the analysis (Trigeorgis, 1999; Copeland and Antikarov, 2001). They explain that the level of uncertainty surrounding an investment project changes due to the changing payouts at the various parts of the decision tree. It therefore requires that the risk-adjusted discount rate should be adjusted to reflect the relative risk associated with the changing situation. Thus, the 17.5% discount rate used for the decision to pre-commit cannot be used when the decision to defer and possibly abandon the project was considered. However, the approach assumes that the risk-adjusted discount rate is constant over the entire life of the project. If decision tree analyses were to be done correctly, the discount rate has to be altered every time a decision could be made, and would have to do so for all the possible outcomes (Copeland and Antikarov, 2001). This may not be practical or could be too complex (Neely and de Neufville, 2001). Thus the DTA technique may value risky projects incorrectly (Trigeorgis, 1999; Copeland and

Antikarov, 2001). According to Trigeorgis (1999), Copeland and Antikarov (2001), one way to correct the deficiencies of the traditional NPV and the DTA is with the use of real option analysis. Although the use of the real option approach is gradually gaining popularity in some investment sectors (e.g., pharmaceuticals), the approach is still under investigation for applicability in other investment sectors such as agriculture.

2.4 Other Traditional Models

This section briefly discusses three other traditional capital budgeting models, namely; the internal rate of returns, the payback period and the accounting rate of return[6].

2.4.1 Internal Rate of Return (IRR)

Generally, NPV decreases as the discount rate increases. At some value of the discount rate, the NPV of a specific cash flow stream is zero. This discount rate that makes NPV equal to zero is called IRR. In other words, IRR is the rate of discount of an investment that equates the present value (PV) of the investment's cash outflows with the PV of the investment's cash inflows (Herbst, 1982). IRR is computed by setting the NPV equal to zero and solving for the discount rate such that the discounted net returns sums to the initial cost of the investment. If the IRR is less than the required rate of return (RRR) targeted by management, then the project would not be undertaken. The setting of the RRR is usually guided by the cost of borrowing for a similar project. However, RRR is usually set at several percentage points higher than the cost of borrowing, to compensate for the risk, time and trouble associated with the investment (Butler and Schachter, 1989).

According to Butler and Schachter (1989), a major criticism of this approach is that it cannot handle unconventional cash flows properly (e.g. negative cash flows coming in later years). If the signs of the net cash flows change in successive periods, it is possible for the calculations to produce multiple IRRs (Brigham and Gapenski, 1997). While multiple rates are theoretically possible, only one rate of return is significant in an 'accept' or 'reject' decision. Also IRR assumes that all funds can be re-invested at the same rate, which is often not possible. Finally, the IRR could lead to a conflicting recommendation when projects are mutually exclusive (Weston and Brigham, 1993). Projects are said to be mutually exclusive when the acceptance of one excludes the acceptance of the other.

2.4.2 Payback Period (PBP)

Also referred to as the capital recovery period, PBP is defined as the length of time required for a stream of cash proceeds produced by an investment to equal to the original cash outlay or the initial investment (Weston and Brigham, 1993). If an investment is expected to produce a stream of cash proceeds that is constant from year to year, the PBP can be determined by the formula below (Butler and Schachter, 1989; Weston and Brigham, 1993):

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[6] For a detailed discussion on these approaches and their limitations, readers are referred to Bierman and Smidt (1984), Butler and Schachter (1989), Brigham and Garpenski (1997) and most corporate finance textbooks.

PBP = Initial cost of investment / Annual Net Cash Inflow

However, if the stream of expected proceeds varies from year to year, the PBP is determined by adding up the proceeds expected in successive years until the total is equal to the initial cost of investment. A project is accepted as worthwhile if the pay back of the initial investment outlay is obtained within a predetermined time. Projects with shorter payback periods rank higher than those projects with longer payback periods.

One major criticism of the payback method is that it does not take into account the time value of money (Herbst, 1982). It also ignores any benefits that occur after the payback period, so a project that returns $50,000 after a five-year payback period is ranked lower than a project that returns nothing after a four-year payback (Weston and Brigham, 1993). PBP methodology favours project having higher cash flows in early years but cash flows in many projects are slow to start for a number of reasons (Butler and Schachter, 1989).

2.4.3 Accounting Rate of Return (ARR)

The accounting rate of return, also known as return on investment (ROI), is an accounting ratio that expresses the profit of an organization before interest and taxation as a percentage of the capital employed. The rate of return of a project is compared with a predetermined rate by management to be the cut-off criterion or the minimum desired rate of return. In this framework, a project is accepted if the ratio exceeds the targeted rate of return.

One of the weaknesses of this approach is that it fails to recognize the time value of money (Herbst, 1982). Since the proposed investment is measured in percentage terms, the ARR method is unable to take into account the financial size of a project when alternatives are compared, and may favour small projects over large profitable ones (Weston and Brigham, 1993). Finally, the target rate of return is an ad hoc risk adjustment (Butler and Schachter, 1989).

2.5 Chapter Summary

This chapter focused on discussing the traditional methods of capital budgeting. Three main methods, namely the static NPV, simulation and decision tree analysis along with their limitations were discussed. There are two main procedures to estimate an NPV of a project. The risk-adjusted discount rate discounts the expected cash flows with a discount rate that includes a risk premium while the certainty equivalent approach discounts the certainty equivalent cash flows with a risk free rate. An implicit assumption of the NPV methodology is that an immediate acceptance or rejection decision needs to be made. However, managerial decisions can be delayed, particularly when there is uncertainty. The simulation approach uses a repeated random sampling technique to determine the risk characteristics of a project's cash flow or NPV based the probability distribution of the underlying variables in the model. The approach has the advantage of dealing with uncertain and complex decision problems but the estimation procedure can be cumbersome. In an attempt to account for the value of managerial decision-making, the DTA lay down an effective structure with all the feasible alternative decisions and the implications of taking those decisions. Other traditional methods that were discussed in this chapter include the internal rate of return, payback period and accounting rate of returns.

Using a simple example, the static NPV and the DTA were compared. The static NPV implicitly assumes no flexibility in decision making and as a result underestimated the value of the project with an NPV of -$6.48. When the analysis considered the flexibility of deferring the decision to pre-commit and the possibility of later abandoning the project when market conditions are unfavourable, the value of the project improved to $23.40 with DTA. However, this approach is also flawed because it assumes a constant discount rate even when uncertainty is changing based on the changing payouts at the various parts of the decision tree. As will be shown later, the real options approach is proposed as a model that corrects both deficiencies. It models the flexibility of decision making by forming replication portfolios, which eliminates theoretical arbitrage opportunities and correctly prices projects with flexibility. The chapter that follows introduces the theory and methods of the real options approach to capital budgeting.

2.6 Tables for Chapter 2

Table 2.1 Cash Flows in the Investment Project

Year	Initial Cost	Cash Flows	Probability
0	$0	$0	
1	$115M	$170M	50%
		$65M	50%

Source: Copeland and Antikarov (2001, pg.88).

2.7 Figures for Chapter 2

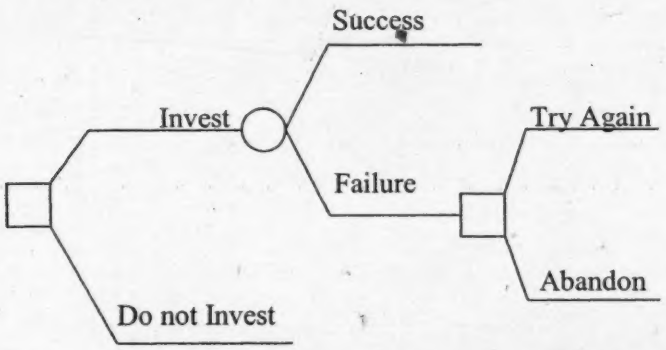

Figure 2.1: A General Structure of a Decision Tree.
Source: Trigeorgis (1999), pg 59

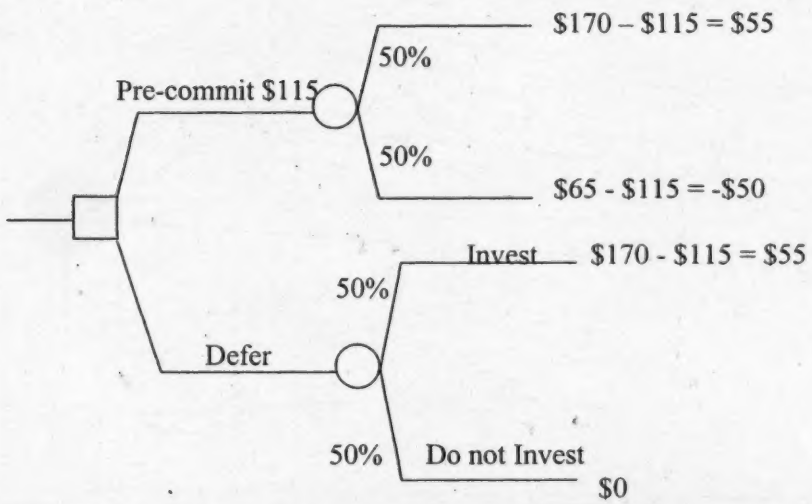

Figure 2.2: The Various Alternative Investment Decision for the Project
Source: (Copeland and Antikarov, 2001, pg 90)

CHAPTER 3:REAL OPTION PRICING THEORY

The development of real options is based on financial option pricing tools and management techniques (Luehrman, 1998). In order to appreciate and comprehend real options, knowledge of financial options will be useful. The section that follows gives a brief overview of the financial (stock) options and their characteristics. A discussion on real options theory, the similarities and differences between real and financial options as well as the different types of real options follow next. The real option valuation methods and the intuition behind the methods are also discussed.

3.1 Overview of Financial Options

An option is a security that gives the holder the right, but not the obligation, to buy or sell a designated asset (e.g. common stock[7]) at a predetermined price on or before a specified date (Trigeorgis, 1999; Hull, 2002). The fixed price in option terminology is called the exercise price and the given date the maturity date. The key property of options is that the holder has the right but not the obligation to fulfil the option contract. This means that if the option is out of the money, the option holder can let the option expire and only lose the money (the premium) that was paid for the option (Cox et al., 1979). An option is said to be out of the money if it has a negative intrinsic value. Intrinsic value is defined as the value of the option when exercised.

3.1.1 Call and Put Options

A call option gives the holder the right to buy the underlying asset at a fixed price, called the strike price, on or before the expiration date of the option. If at expiration, the value of the asset is less than the strike price, the option is not exercised and it expires worthless. If, on the other hand, the value of the asset is greater than the strike price, the option is exercised.

A put option, on the other hand, gives the holder of the option the right to sell the underlying asset at a fixed price, also called the strike price, on or before the expiration date of the option. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If on the other hand, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option.

There are two sides to every option contract; long and short positions. A long position involves the holding or purchasing of the option while a short position involves the selling or writing of the option. The writer of an option receives cash up front but has potential liabilities later. The profit or loss of the writer is the reverse of that for the purchaser of the option.

There are four types of options positions (Hull, 2002): 1) A long position in a call option, 2) A long position in a put option, 3) A short position in a call option, and 4) A short position in a put option. Assuming X is the exercise price and S is the price of the underlying asset, the intrinsic values or payoffs of the options are presented in Table 3.1. For a call option, the option will be exercised if the stock price (S) is greater than the

[7] The discussion in this section considers common stock as the underlying asset. Hence, underlying asset and stock is used interchangeably.

exercise price (X). The reverse is the case for a put option. Puts are exercised if X is greater than S.

Financial options are further divided into two different kinds, depending on their ability to be exercised. If the option can be exercised at any time up to the expiration date it is called an American option and if the option can only be exercised at the expiration date it is referred to as an European option. The possibility of early exercise makes American options more valuable than otherwise similar European options, particularly when the underlying asset pays dividends. It also makes these options more difficult to value (Hull, 1997). The following section describes the factors that determine option values.

3.2 Determinants of Option Values

The value of an option is determined by a number of variables relating to the underlying asset. There are six factors that affect the value of an option (Hull, 2002). These are the current price of the underlying asset, the exercise price, the time to maturity, the risk free interest rate, the volatility (standard deviation of returns of the underlying asset), and the dividends expected during the life of the option. Table 3.2 summarizes the factors and their predicted effects on call and put values in standard financial option models. The effect indicates a change in one factor holding all other factors constant.

Options are assets that derive their value from the underlying asset. Consequently, changes in the value of the underlying asset (i.e., stock price) affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increases.

Another key factor that affects the value of an option is the exercise price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.

Both American call and put options become more valuable as the time to expiration increases. This is because the longer time to expiration provides more opportunity for the value of the underlying asset to move, increasing the value of both types of options. European call and put options do not necessarily become more valuable as the time to expiration increases (Hull, 2002). This is because the owner of a long-life European option can only exercise at the maturity of the option.

An increase in the risk-free interest rate will have a positive effect on a call option since the exercise price will only have to be paid at the maturity date, making it possible to invest money elsewhere. The opposite is the case for a put option since the proceeds from the sale of the underlying asset are received in the future.

The volatility of the underlying asset value is a measure of uncertainty in the future movements of the asset's value. Option values are very sensitive to the volatility of the underlying asset value. A higher volatility is always positive for both call and put options in standard financial option models. This is because it widens the distribution of the future asset's value and hence the possible intrinsic value of the option if exercised. For instance, the value of a call option depends on the value of the asset exceeding the exercise price. As the distribution of the future asset widens due to increase in the

uncertainty of the project value, the expected payoff conditional on the option expiring in the money also increases.

Finally, the value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a decreasing function of expected dividend payments, and the value of a put is an increasing function of expected dividend payments.

As indicated at the beginning of the chapter, the coverage of the financial option theory in this thesis is very brief. However, the theory of financial option pricing has been studied extensively in both academic and trade literature. For a detailed discussion on this topic, interested readers are referred to Hull (1997, 1998, and 2002), Kolb (2000), Ross et al. (1999) Brigham and Gapenski (1997), Brealey and Meyers (1991), Weston and Brigham (1993) and many other finance textbooks. The remainder of this chapter explores the real options literature and methodology.

3.3 Real Options Theory

The real options theory is the application of financial option pricing principles and management techniques to value investments in real assets (Amram and Kulatilaka, 1999). A real asset is usually tangible such as a business unit or a project compared to financial assets, which typically consist of stocks, bonds, currency, etc (Copeland and Antikarov, 2001). Like financial options, real options involves the right, but not the obligation, to take an action (e.g., defer, abandon, expand, etc.) at a predetermined cost called the exercise price for a predetermined period of time called the time to maturity or the life of the option (Reiss, 1998). Real option theory provides a framework to analyse strategic capital investments by enabling investments to be treated as opportunities rather than now or never type investment decisions (Dixit and Pindyck, 1995). With the real option methodology, decision-makers could keep investment options open when facing uncertainty and undertake them after resolving uncertainty with time or more information (Trigeorgis, 1993a, 1999). The real option approach has the potential of quantifying the value of active management decisions. This value is usually manifested as a collection of real options, either call or put, embedded in an investment opportunity (Trigeorgis, 1993a). Some of these real options occur naturally without incurring an up-front cost (e.g. defer, abandon) or, at some additional expense, may be planned and built into an investment opportunity (e.g. expand, switch) (Kulatilaka, 1993; Trigeorgis, 1996). Building real options into an investment opportunity may be preferable if the present value of the cost of making changes at a later date is greater than the additional cost of designing flexibility into the investment opportunity at the outset (Lander and Pinches, 1998).

The difference between the static NPV analysis and the real option analysis results from the different impact some of the variables have on the models. With the real options principle, higher uncertainty, greater interest rates, or more time before undertaking a project does not necessarily make an investment opportunity less valuable as is the case with the NPV model (Trigeorgis, 1999). As can be seen from Table 3.3, each of these factors have a negative effect on the traditional NPV of an investment opportunity (downward-pointing arrows in Table 3.3). These factors may however enhance managerial flexibility and hence the option value (upward pointing arrows).

3.3.1 Real Options and Financial Options - Similarities

As indicated earlier, real options analysis is based on the conceptual framework of financial options. An opportunity to invest in a real asset bears similarity to an option to invest in a financial asset. Both involve the right, but not the obligation, to acquire an asset by paying a certain sum of money on or before a certain time. For example, a call option gives the buyer the right, but not the obligation, to buy a security at a specified price in the future. Similarly, a capital investment today that gives the investor the future right, but not the obligation, to make a further investment is a real option. Like financial options, five main variables determine the value of real options, plus a sixth variable that is very important, particularly when evaluating American options. A close analogy can be made between the variables that determine the value of a financial option and a real option on a project investment. Table 3.4 shows a comparison of variables use to value the two types of option.

Most investment decisions share three characteristics in varying degrees that are common to both financial and real options and hence important for the analogy between the two options (Dixit and Pindyck, 1994). These characteristics are irreversibility, uncertainty, and managerial freedom and timing. These factors play a key role in determining the optimal decisions of investors.

3.3.1.1 Irreversibility

Dixit and Pindyck (1994; 1995) claim that in many investments in both financial and real assets, the initial investment outlay may be partially or completely irreversible and is more or less a sunk cost for the investor. There are totally irreversible investments where the whole investment cost is lost after it has been incurred, and there are partially irreversible investments whose value can be partially recovered (Dixit and Pindyck, 1994). For example, if an investor invests in a pork project that shows itself to be unfavourable, the investor will end the project and try to recover as much as possible but may not be able to recover all the initial cost put into the project at the outset.

An irreversible investment opportunity is much like a financial call option. A financial call option gives the holder the right, for some specified amount of time, to pay an exercise price and in return receive an asset (e.g. a share of stock) that has some value. Exercising the option is irreversible and the investor cannot retrieve the option or the money that was paid to exercise the option. Equally, a firm has an option to invest in for example, a project at a specific cost (the exercise price) in return for a future asset of some value. The asset could be sold to another investor, but the investment is irreversible (Dixit and Pindyck, 1994).

3.3.1.2 Uncertainty

Future rewards in many financial and capital investments are uncertain. For example, the future demand for an agricultural biotechnology product developed in a specific R&D project will be high or low, in relation to the forecasted demand of the product. The investor then assesses the probabilities for alternative outcomes that can mean greater or smaller profit for the project. A major similarity between a financial call option and a firm's option to invest in a specific project is that the value of the option is in part due to the fact that the future value of the asset obtained by investing is uncertain.

Two types of uncertainty are present in most capital investments; economic uncertainty and technical uncertainty (Dixit and Pindyck, 1994).

Economic uncertainty is an exogenous uncertainty that is present in the market place (e.g., interest rate, inflation, industry prices, cost movements etc.), and is related to factors that cannot be influenced by the actions or decisions of management. The implication is that economic uncertainty is generally correlated with the actual movement of the general economy. Management is not passive to these changes and will instead revise investments and operating decisions in response to market conditions, in order to maximize the company's wealth, but management cannot affect the economic movements. Technical uncertainty, on the other hand, is a project specific uncertainty and is not correlated with the general movements in the economy. This type of uncertainty is endogenous to the decision process and is affected by the decisions of management. For example, it is only possible to reduce the uncertainty in the outcome of an R&D project with an actual step-by-step investment strategy where each step provides valuable information, which reduces the uncertainty.

3.3.1.3 Managerial Freedom and Timing

In most financial and capital investment opportunities, there is managerial flexibility embedded in the projects and the higher the degree of the managerial freedom, the better, and the higher the value of the investment opportunity. This means that embedded options can only add to the value of an investment (Sharp, 1991). Two types of flexibility could be present in a project, internal and external flexibility. The internal flexibility is the managers' flexibility in the project itself. This is the flexibility to modify the project as the future conditions change. These changes can include expansion, alteration, and even abandonment. The external flexibility is the growth option, which gives the possibility to be able to invest in another project that may not have been possible originally (Flatto, 1996).

3.3.1.4 Dividends

Another similarity between real and financial options analogy is how dividend payments are accounted for when valuing both options. In the case of financial options, (Hull, 1998) content that if an underlying stock of a financial option pays dividends, then the option is on a dividend paying stock. Therefore the valuation of the option should be adjusted to account for the effects of the dividends on the option values. This is because dividend reduces the value of the underlying stock and therefore affects the value of the option contingent on it.

Similarly, if the underlying asset of a real option pays dividends, the value of the asset decreases by the magnitude of the dividends paid at the ex-dividend date. Amram and Kulatilaka (1999) refer to dividend payments as leakage in the underlying asset value. Many real assets experience leakage in value in the form of dividend payments, which affect the value of the option contingent on the asset and the timing of the optimal investment decision. This requires an adjustment to be made when valuing a real option of a dividend-paying asset to account for the impact of the dividends on the value of the option.

3.3.2 Real Options and Financial Options – Analogy Limitations

The analogy between financial and real options has its limitations. According to Kester (1993) and Trigeorgis (1999), there are three potential factors that make real options different from financial options. These factors are discussed below.

3.3.2.1 Proprietary or Shared

Financial options are "proprietary" in that it gives its owner an exclusive right of whether and when to exercise, i.e., the holder does not have to worry about competition for the underlying asset. This could also be the case for the holder of a real option. This is the case when a company has a unique know-how in a technological process or has a patent right. These real options have a proprietary characteristic since the company has the exclusive right to exercise these options.

The limitation of real option in the analogy is that some investment opportunities are not proprietary, in that they may be jointly held by more than a single competitor (Smit and Ankum, 1993). Trigeorgis (1999) refers to these kind of real options as "shared real options" since they could be exercised by any of the participants. The choice to introduce a new product that is not protected from a possible introduction of close substitutes by a competitor or the chance to enter a new market with no barriers to competitive entry are examples of shared real options. All things being equal, shared real options are less attractive than proprietary ones because competitors can make counter investments that can erode the profitability of a company (Kester, 1984; Smit and Ankum, 1993).

3.3.2.2 Simple or Compound

Standard financial option's value is derived only from the underlying asset. Trigeorgis (1999) calls these kinds of option as "simple" because their exercise depends on the value of the underlying asset at the maturity date. Similarly, some real options are "simple" in that their value upon exercise is limited to the gross present value of the underlying project (Kester, 1984).

Some real options are however more complex in character. When a real option gives the holder the right not only to receive the underlying asset but also to receive further investment opportunities in the future, the option becomes more complex. In essence, they are options on options (i.e. options whose payoff is another option) (Trigeorgis, 1999; Copeland and Antikarov, 2001). These kinds of options are referred to as compound real options. Investments in R&D or a lease on an undeveloped tract with potential oil reserve are examples of these types of options. The problem with these options is that they cannot be seen as individual options and as such cannot be added together (Trigeorgis, 1993b). The procedure to calculate the values of compound options can be complex and cumbersome (Trigeorgis, 1999).

3.3.2.3 Non-tradability

Financial options are written on traded securities, which make it easier to determine their parameters. The security price is usually observable and as a result the variance of its rate of return can be easily estimated from historical data or by using the implied variance of similar options on the same security (Copeland and Antikarov, 2001). However, real options like most investment projects, are not generally tradable. Some of

the proprietary real options, such as investment opportunities related to patents and licensing agreements, may be traded but at a substantial cost in imperfect markets (Trigeorgis, 1999). On the other hand, shared real options may not be tradable at all on the market because they may already be a public good for the whole industry (Smit and Ankum, 1993). This makes it difficult to estimate the variance of the rate of return of the underlying asset since some projects are very unique and do not have any historical data source (Copeland and Antikarov, 2001). One way to get round this problem is by using management's subjective estimates.

3.4 Types of Real Options

Trigeorgis (1999) and Copeland and Antikarov (2001) identify various types of real options that can provide management with valuable operating flexibility and strategic adaptability. These types of real options are explained below.

3.4.1 Option to Defer

Projects that have to cope with irreversibility and economic uncertainties could increase in value when new information becomes available and uncertainty is decreased as time goes by (Dixit and Pindyck, 1994). The option to defer occurs when a decision can be put off until some date in the future. This allows management to determine if resources should be spent on a project at that future date or not over time. The option to defer enables management to benefit from improved market conditions. The value of waiting to invest is analogous to the value of an American call option on the gross present value of the project's future expected cash flows (Trigeorgis, 1999). This option is more valuable when market conditions are unstable and with long investment horizons. Investment will be carried out (exercise the option) only if the project will be beneficial else management will not commit to it. The option to defer is particularly valuable in resource extraction industries, farming, paper products, and real estate development (Trigeorgis, 1996).

One issue that has been raised in the real options literature is the question of how valuable is the option to defer when market is competitive. Trigeorgis (1991; 1999), Smit and Ankum (1993) and Leahy (1993) have examined the impact of competitive entry on the value of the option to defer. Trigeorgis (1991; 1999) and Smit and Ankum (1993) found that the impact of competitive entry could be treated analogous to American call option on a dividend-paying asset. The impact of this is to increase the possibility of early exercise. Leahy (1993), on the other hand, concluded that the introduction of competition does not affect the timing of investment decisions when the investment is irreversible.

3.4.2 Compound Options

Compound options are options whose value is contingent on the value of other options. Compound option can be either simultaneous or sequential. Simultaneous compound option occurs when the underlying option and the option on it are simultaneously available (Copeland and Antikarov, 2001). For instance, the equity of a leveraged firm is a call option on the value of the assets of the firm with an exercise price equal to the face value of the firm's debt and whose maturity is the maturity of the debt. Therefore, a call option on the equity of a firm is an option whose underlying asset (the equity of the firm) is itself an option. This is a simultaneous compound option because

the equity of the firm (a call option) and the call option on the equity are alive simultaneously.

A sequential compound option, on the other hand, occurs when a project investment happens in a series of outlays. In most real life projects, the required investment may not be incurred as a single up-front outlay, instead as a series of outlays over time. The staging of the capital investment as a series of outlays over time creates valuable options to default at any given stage (Copeland and Antikarov, 2001). Thus each stage can be viewed as an option on the value of the subsequent stages by incurring the cost outlay required to proceed to the next stage and can therefore be valued as options on options (Trigeorgis, 1999). The key characteristic of sequential investments is the ability to temporarily or permanently abandon the investment if the value of the completed project falls. The possibility of stopping midstream could be looked at either as a call option where the investor has the option to invest in the next stage of the investment or as a put option where the investor has the possibility to stop and save the cumulative losses of the future. When viewing the staged investment as a call option it is similar to options on options because the completion of one stage gives the right but not the obligation to undertake the next stage and the options that this stage provides. This option is valuable in R&D intensive industries like pharmaceuticals, venture capital financing, natural resource extraction, among others (Trigeorgis, 1999).

When a compound option is characterized by multiple sources of uncertainty, it is referred to as a rainbow compound option (Copeland and Antikarov, 2001). In other words, rainbow options are those that are driven by several sources of uncertainty. Valuing compound rainbow options can be complex since the uncertainties are usually kept separate[8].

3.4.3 Option to Alter Operating Scale

The option to change scale can result in the project being expanded, contracted, or shut down and restarted. Depending on market conditions that prevail at a particular time, the rate of resource expenditure can be adjusted to meet the new conditions.

3.4.3.1 Option to Expand

If output price or market conditions turn out to be more favourable than expected, the scale of operation of a project can be expanded by incurring an additional cost. This is analogous to a call option to acquire an additional part of the initial project where the cost to expand is the exercise price. This managerial flexibility has a value and the cost of expanding could often be reduced if flexibility is built into the project at an early stage (Trigeorgis, 1999). The option to expand may also be of strategic importance, especially if it enables the firm to capitalize on future growth opportunities. This type of option is typically found in the natural resource industry (Trigeorgis, 1999).

3.4.3.2 Option to Contract

The option to contract has a positive value if market conditions turn weaker than originally expected, management can then reduce the scale of operations and thus saving part of the planned investment outlays. This flexibility to mitigate loss is analogous to a

[8] Examples of how rainbow compound options are valued using the binomial model are presented in Copeland and Antikarov (2001).

put option on part of the initial project, with exercise price equal to the potential cost savings (Trigeorgis, 1999). This may be particularly valuable in the case of new-product introductions in uncertain markets.

3.4.3.3 Options to Shut Down and Restart Operation

The managerial flexibility to be able to shutdown and restart operations can be valuable if prices are such that cash revenues are not sufficient to cover variable operating costs. It might be better not to operate temporarily. If prices rise sufficiently, operations can be restarted. Thus, operations in each year may be seen as a call option to acquire that year's cash revenues by paying the variable costs of operating as exercise price (Trigeorgis, 1999). It is equivalent to the firm having a portfolio of call and put options. For example, being able to temporarily shut down a project is equivalent to a put option and restarting operations when the project has been down is equivalent to a call option. This option must however be exercised with care since it could lead to erosion of valuable expertise that could be used elsewhere in the business (Trigeorgis, 1999).

3.4.4 Option to Abandon

The option to abandon allows a company to abandon a project if the market conditions drop dramatically. Management may have a valuable option to abandon the project permanently in exchange for its salvage value or halt operating losses. This option can be valued as an American put option on the project's current value with an exercise price equal to the salvage value. Like the option to shut down and restart operation, this option should not be exercised without taking into consideration the costs, if it might lead to eventual loss or erosion of valuable expertise and other crucial organizational capabilities that could be applied elsewhere in the business or that could prevent the firm from participating in technological development (Trigeorgis, 1999). The option to abandon a project provides a partial insurance against failure. Abandonment options are important in capital-intensive industries where the possibility of capturing some (resale) value for assets is high.

3.4.5 Option to Switch

The option to switch allows an organization to change either the input mix or output mix of a facility if prices and demand conditions change. It provides valuable built-in flexibility to switch from the current input to the cheapest future input, or from the current output to the most profitable future product mix, as the relative prices of input or output fluctuate over time. In fact, the firm should be willing to pay a certain positive premium for a flexible technology that can change the inputs from expensive to cheap and change the output from cheap to expensive, depending on the market. Process flexibility can be achieved not only via technology (e.g. by building a flexible facility that can switch among alternative energy inputs), but also by maintaining relationships with a variety of suppliers and switching among them as their relative prices change. This type of option is more valuable in industries such as automobiles, electronics, pharmaceutical, and oil industries (Trigeorgis, 1999).

3.4.6 Growth Option

Growth options that set the path of future opportunities are of considerable strategic importance (Trigeorgis, 1999). The growth option provides the company that invests in a specific project with a possibility to make a follow-on investment in the future and is analogous to a call option. The option to grow is used when an initial investment is required for further development. The project can be considered as a prerequisite or links in a chain of interrelated projects and may serve as a springboard for future project generation. But unless the firm makes that initial investment, subsequent generations will not be feasible. An early investment in, for example, an R&D or strategic acquisition is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities (Trigeorgis, 1999). This type of option is valuable in industries with high technology in R&D, venture companies, and long-lived projects with several phases.

3.4.7 Multiple Interacting Options

Real life projects are often more complex in that they usually involve a collection of multiple sequential real options, whose value may interact (Kulatilaka, 1993). Upward potential enhancing and downward protection options could be present in combination for a particular project. As a result, the combined value of two or more options in the presence of each other may differ from the alternative of evaluating each option separately and adding their values. It could be higher when there are positive interactions among the options or lower when interactions are negative. According to Trigeorgis (1993b), the incremental value of an additional option, in the presence of other options, is generally less than its value in isolation, and the value declines as more options are present.

The traditional capital budgeting approach of evaluating investments, such as the NPV, often fails to adequately capture the value of the above-discussed managerial operating strategies. For instance, the flexibility to optimally time project initiation is often ignored by NPV, because it implicitly assumes that immediate acceptance and rejection decision needs to be made (Trigeorgis, 1991). However, the ability to wait and see could be valuable to managers, especially for projects that are present in an uncertain environment such as investments in agriculture. Similarly, the traditional NPV approach may fail to properly address project valuation of growth opportunities or other strategic alternatives (such as abandon or expand) that arise from investment projects over time. The above shortcomings of the traditional approach necessitated the use of the of the real options approach to capital budgeting. This is because the options approach has the potential to include the value added to a project from strategic management (Copeland and Antikarov, 2001). The section that follows discusses some of the real options application in the literature.

3.5 Real Options Applications

Research in real options has mostly focused on valuing the different types of options discussed above. In this section, a discussion of some of the types of real options that have been valued and the researchers that valued them is presented.

The most commonly valued real option is the option to defer (Trigeorgis, 1999). It is usually found in the natural resource industries, where commodity prices play a

significant role in industry profitability. Tourinho (1979), McDonald and Siegel (1986), Ingersoll and Ross (1992), Bjerksund and Ekern (1993), Kemma (1993), Cortazar and Schwartz (1997) and Paddock, Siegel and Smith (1987), Price and Wetzstein (1999), Bailey and Sporleder (2000), Maung and Foster (2002), have all looked at valuing this option. Bjerksund and Ekern (1993) compared the range of values within an offshore oil field development for everything from immediate irreversible investment to different degrees of flexibility with regard to the timing of the investment and the option to abandon. They introduced oil prices as the only source of uncertainty where the price process was the geometric Brownian motion price process. Paddock, Siegel and Smith (1987) compared valuation made via discounted cash flow (DCF) and real option analysis on a new undeveloped oilfield against actual bids made on oil leases. They valued the option as a compound option with the option to delay or abandon at each stage. The option valuation proved to be much closer to the ultimate industry bid than the standard DCF technique. In the field of agriculture, Price and Wetzstein (1999) looked at the optimal entry and exit threshold for Georgia commercial peach production when both price and yield follow a Brownian motion process. Bailey and Sporleder (2000) considered the opportunities for value added production through further processing of a new generation cooperative. The investment was modeled as a sunk cost and the results show that flexibility of managers was an important criterion for making decisions regarding sunk cost investments.

Compound sequential options are very important in long-term investments where capital outlay may be staged. Each stage represents an option on subsequent stages where the project could be abandoned at any stage. Carr (1988), Trigeorgis (1993b), Paddock, Siegel and Smith (1988), Kulatilaka and Trigeorgis (1994), Ott and Thompson (1996) have valued this type of option. Paddock, Siegel & Smith (1988) valued an option as a compound option where the exploration expenditures were the exercise price to obtain undeveloped reserves. Development expenditures enabled one to convert undeveloped reserves into developed reserves. Extraction expenditures were necessary to extract the oil and ultimately sell it. The option to delay or abandon was incorporated at every stage of the project.

Brennan and Schwartz (1985), McDonald and Siegel (1985), Kemma (1993) and Trigeorgis and Mason (1987) have examined the option to expand or contract and shut down and abandon. These options could be exercised in response to changes in output prices or changes in the business cycles. In Brennan and Schwartz's (1985) article, the scale of operation of the project was assumed to vary with the output prices of the commodity (copper in this case). They also explicitly accounted for the cost of shutting down and starting up should the price of copper hit a threshold price. The threshold prices were determined by holding output prices constant and valuing both the operating mine and the option to close concurrently to find the cross over points where strategic decision should be made. Trigeorgis and Mason (1987) in addition to looking at the value of altering operating scale examined the value of growth options.

The option to switch inputs and/or outputs is common in most industries. If product or commodity prices change, or if demand changes, managers have the flexibility to switch to a more favorable product or a cheaper input. On the output side, consumer product companies have the option to switch between products according to which is currently more favorable. For the input side, management can choose between barley and

corn as the main component of feed ration in hog production for example. Authors like Kensinger (1987), Kulatilaka (1993), and Kulatilaka and Trigeorgis (1994), Baldwin and Ruback (1986) have evaluated this option. Kulatilaka (1993) examined the flexibilities in the case of a dual fuel boiler as opposed to a single fuel boiler. The option to switch and potentially switch back was viewed as a nested compound option, a call on gas prices and a put on the oil price. In their example the flexibility of switching with relatively low switching costs added approximately 40% more value to the boiler, which was well in excess of the additional cost of the dual fuel boiler. Baldwin and Ruback (1986) showed that future price uncertainty creates a valuable switching option that benefits short-lived projects.

Growth option occurs when an initial investment is required for further development. The project can be considered as a prerequisite or links in a chain of interrelated projects. Each project in the link is required for the future growth. Myers (1977), Kester (1984, 1993) Trigeorgis and Mason (1987), Pindyck (1988), Trigeorgis (1998), Smith and McCardle (1998) have all valued this option. Smith and McCardle (1998) integrated option pricing with decision analysis in order to value oil developments and the growth inherent in them. They develop a model where prices and production rates vary stochastically and apply it to an actual property in the Permian basin in Texas. They then looked at options related to increasing production via the drilling of additional wells and the temporary suspension of production.

3.5.1 Multiple Options

Despite the enormous theoretical contribution, the focus of earlier research was valuing single real options (i.e. one option at a time). However, real-life projects are often more complex in that they involve a collection of multiple options whose value may interact (Trigeorgis, 1993b). An early exception is Brennan and Schwartz (1985), who determined the combined value of the option to shut down and re-start a mine, and to abandon for salvage value. They recognized that partial irreversibility resulting from the cost of switching the mine's operation state may create a persistence, inertia or hysteresis effect, making it optimal in the long-term to remain in the same operating state even though short-term considerations (i.e., current cash flows) may seem to favour an immediate switch. Trigeorgis (1993b) and Kulatilaka (1993) have also examined option interactions and found that many of these interactions offset each other thereby reducing total option value and thus simply adding up the options valued in isolation is likely to overstate project value. Trigeorgis (1993b) focused on the nature of real options interactions, pointing out, for example, that the presence of subsequent options can increase the value of the effective underlying asset for earlier options, while exercise of prior real options may alter the underlying asset itself, and hence the value of subsequent options on it. Thus the combined value of a collection of real options may differ from the sum of their separate values.

3.6 Critique of the Real Options Approach

The real options literature to date has provided many insights into capital budgeting decision-making and produced a new or enhanced framework for modeling and valuing investment opportunities having real options. These insights include valuing managerial flexibility and providing guidelines on when to exercise the options inherent

in real investments. However, many of the real option analyses presented to date are relatively straightforward or have been stylized. Yet, from a practical standpoint, real option modeling is not easy to use or implement as real world investment opportunities are often sophisticated and complex. As a result, the interest in real options has been mainly academic and the practical implementation has so far been quite limited (Lander and Pinches, 1998; Davis, 1998).

The mathematics behind the option pricing formulas is quite complex and it is hard for corporate managers, practitioners, and even academics to get a deeper understanding of the valuation techniques (Lander and Pinches, 1998). This makes it difficult for the technique to be easily incorporated into a company's existing capital budgeting framework since managers are unwilling to base their decision on analysis they do not understand. Thus the application of the options approach usually requires a leap of faith to move from a familiar, though somewhat flawed system, to the unfamiliar. This is one of the major disadvantages compared to the more intuitive traditional techniques.

Also to find the correct input variables to the option pricing formulas could be difficult and especially to find the appropriate volatility in the underlying project. It might be known that the specific project is risky but to find an asset or portfolio of assets that are correlated with the project and that are traded in the financial market could be quite difficult. One way of getting around this problem is by assuming that the asset is correlated with itself and as a result the NPV set up of the project can be used to determine the volatility of the project. Copeland and Antikarov (2001) call this assumption the Market Asset Disclaimer.

Again the real option approach is based on the principle that by forming a replicating portfolio, the possibility of theoretical arbitrage profit would be eliminated. However, real life capital projects cannot really be replicated.

Many managers see the results from the option valuation as something that comes out of a "black-box" (Faulkner, 1996). This view of the option theory as being a strange black-box that is hard to understand is one of the major barriers to the acceptance of the new option valuation theories (Nichols, 1994).

3.7 Real Options Valuation

The valuation techniques of real options and its similarity to the valuation of financial options are one of the major fundamentals of the option approach to capital budgeting. The possibility of being able to use the financial option valuation techniques when valuing real assets is based on an important assumption that an option can be replicated by a portfolio of traded securities. Since this equivalence is not dependent on risk attitudes, the value of the expected future payoffs can be derived from a risk-neutral approach and discounted at the risk-free interest rate. If this had not been the case there would be arbitrage and "free lunch" possibilities. This section discusses the two most common and widely used real options models namely; the Black-Scholes and the binomial models (Trigeorgis, 1999). The strengths and limitations of each method and the justification for using the binomial model to analyze the case study presented in this thesis are discussed. The different input variables needed to value real options and how they are determined are discussed next.

3.7.1 Determining the Input Variables for Real Options

In order to value real options, there is the need to gather the necessary input variables. As mentioned earlier, the value of real options depends on five main variables plus an important sixth. These variables and how they are determined is the focus of discussion in this section.

3.7.1.1 Gross Present Value of the Project (V)

The gross present value of the project or the value of the underlying risky asset is the present value of the expected cash flows to be received from the project. The value does not include the initial capital requirements for the setting up of the investment project. This variable is determined from the NPV without flexibility set up of the project. To estimate the variable, the expected net cash flows of the project are discounted to the present using an appropriate discount rate for the project. The expected net cash flows consist of all the revenues and expenditures (except the initial capital cost) generated from the investment. To illustrate how this variable is determined, a simple hypothetical numerical example is used. Assuming there is a project that will generate total revenue of $1,200 per year for the next five years starting from year one. The total variable and fixed costs per year for the project are assumed to be $500 and $200 respectively. The annual free cash flows for this example are presented in Table 3.5.

Assuming a risk adjusted discount rate of 10%, the gross present value of the project is estimated as follows:

$$V = \sum_{t=1}^{5} \frac{E(c_t)}{(1+k)^t} = \frac{500}{(1.1)^1} + \frac{500}{(1.1)^2} + \ldots\ldots\ldots\ldots + \frac{500}{(1.1)^5} = \$1895.40 \qquad (3.1)$$

where $E(c_t)$ is the expected free cash flows and k is the risk adjusted discount rate. As can be seen from the example, the gross present value for this project is $1895.40. Note that the procedure for estimating the gross present value is the same as the first half of the NPV model presented in chapter two (equation 2.1).

3.7.1.2 Initial Capital Cost (I)

The value of the investment outlay is equivalent to the exercise price for a financial option. The initial capital cost is the present value of the start-up cost or the lump sum cost incurred at the beginning of a project. Like the gross present value, the initial capital cost is also determined from the static NPV model. The entire cost may be incurred at the beginning of the project or could be incurred over a certain time period. Where the initial investment cost is incurred over a certain time period, future costs have to be discounted to present. To continue from the previous example, assuming it will cost $1800 over two years ($1000 immediately and the remaining $800[9] in next year) for the chance to receive the $500 free cash flows, the initial cost of the investment is estimated as follows:

$$I = 1000 + \frac{800}{(1.1)^1} = \$1727.27 \qquad (3.2)$$

The calculation of the initial cost of the investment also represents the second part of the NPV model presented in chapter 2 (equation 2.1). Thus, the above two variables

[9] Note that the risk-adjusted discount rate is used to discount the investment in year 1. This is because the $800 is expected cost in year 1 but it is assumed to be uncertain.

for the real option analysis can be determined from the NPV model without using any complex mathematical computations or making any additional assumptions other than those used for the NPV analysis.

3.7.1.3 Time to Maturity (T)

The time to maturity for a real project is the time left until the option disappears. For a project this time is generally not as pre-defined as for a financial option and could sometimes be tricky to specify. The time to maturity, in some cases has to be subjectively defined by management as the time it takes for competitors to exploit the same opportunity, whereas in some cases the time to maturity is fixed in advance.

3.7.1.4 Volatility of the Project Returns (σ)

This standard deviation of returns of a project is the most difficult variable to estimate. For a security traded in the financial market it is relatively easy to find the standard deviation by analyzing historical volatility of the returns or by using the implied volatility of the option. For a specific project it is more complex to determine the correct volatility. However, the volatility of projects is usually determined by either evaluating the volatility of a twin security (alternatively a dynamic portfolio of securities), which correlates with the project (Trigeorgis, 1996), or have management subjectively estimate the volatility (Sharp, 1991; Copeland and Antikarov, 2001).

Another approach that can be used and which is adopted for this analysis is the use of Monte Carlo simulation on the project's static NPV model to determine the volatility of the project using a historical data (Copeland and Antikarov, 2001)[10]. The approach is based on the assumption that the future will be like the past. As a result, historical data can be used to predict the future pattern of the uncertain variables in the project's NPV without flexibility model. This requires the identification of the main uncertain variables in the project's NPV. Having identified the uncertain variables (e.g., price, cost, quantity, etc) in the NPV model, the best stochastic process (e.g., mean reversion, geometric Brownian motion, etc.) that fits the historical data is then used to forecast the expected future values of the variables using a simple regression. The standard deviation of the error terms from the regression is used to define the level of uncertainty for each uncertain variable for the Monte Carlo program. When there is more than one uncertain variable, the correlation between the variables is accounted for using the correlation coefficient of the error term between the variables estimated from the regression. The stochastic variables are then incorporated into the static NPV model to calculate the rate of returns of the gross present value of the project. Using any simulation software (e.g., @Risk), Monte Carlo simulation within the NPV model is then run on a number of iterations to calculate the standard deviation of the rate of returns, which is the volatility of the project value.

3.7.1.5 Risk-Free Rate of Return (r)

This variable is the risk-free interest rate for a risk-free bond with the same expiration date as the option being evaluated. It is usually ideal to consider this variable

[10] For now, the general estimation procedure is discussed. The detailed explanation of the approach and how it is applied to this case study is presented in Chapter 5.

after the time to maturity variable is known since it is normally derived from government bonds or treasury bill with the same maturity as the option (Perlitz et al., 1999).

3.7.1.6 Dividend (q)

The sixth variable is the dividends that may be paid out by the underlying asset. Amram and Kulatilaka (1999) refer to dividend payments as 'leakage' in value arising from cash flows that accrues between decision points. The sources of leakage in value include dividends, loss of value through competition, royalty and licensing fees, loss from perishable damage, etc. Many real assets experience leakage in value, which changes how the value of the underlying asset evolve and hence affect the value of the option and the timing of the optimal decision. Unless the value of the leakage of the underlying asset value is accounted for the option may not be correctly valued. Another important effect is that with no leakage of the underlying asset's value, it would not be optimal to exercise American call options such as to wait or expand before its maturity date (Merton, 1973; Copeland and Antikarov, 2001). This is because if the underlying asset pays dividend, its value will be reduced and may therefore become optimal to exercise the option (a call for example) just before or after the dividend is paid. The holder of an option (e.g., management or owner of the underlying) will wait till close to maturity before deciding on whether to exercise if there is no dividend (Trigeorgis, 1991; Hull, 1997). Despite the potential impact dividends could have on option value, its importance has not been adequately addressed in the real options applications. Particularly, the author is not aware of any real option application in agricultural investments that accounted for the effects of dividend on the option values and how it affects the decision of management.

3.8 Real Options Valuation Models

There are several option pricing methods that can be used to value real options. However, this section discusses the two most commonly used real options valuation models namely; Black-Scholes model (a continuous approach) and Binomial model (a discrete approach)[11]. The intuition behind each approach and the justification for adopting the binomial model for evaluating the pork case study are also presented.

3.8.1 The Black-Scholes Model

A commonly used model to value European options is the formula developed by Fisher Black and Myron Scholes in 1973 (Hull, 1997)[12]. The formula gives the price of a European call or put on the basis of two changing variables; the underlying asset value, and time to expiration. The other variables, volatility of the underlying asset, the exercise price, and the risk-free interest rate affect the option's value but are constant. The models for calculating a call option, c, is given as (Trigeorgis, 1999; Copeland and Antikarov, 2001):

$$c = VN(d_1) - Ie^{-rT}N(d_2) \qquad (3.1)$$

[11] Note that these models are basically financial option models but can be used to value real option because of the analogy presented earlier. In this section, the models are presented in real options context.

[12] A few of the articles that have applied the Black-Scholes model include Trigeorgis (1991), Chen et al (1998), Lint and Pennings (1998), Perlitz et al. (1999), Bailey and Sporleder (2000).

where

$$d_1 = \frac{\ln(V/I) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The notations in the model are defined as follows:
V = the gross present value of the underlying asset
I = the initial capital cost or the strike price
T = time to expiration
r = the risk free rate of return
σ = the volatility of rate of returns of the underlying asset
$N(d)$ = the cumulative standard normal density function.

The Black-Scholes model is based on the following assumptions (Black and Scholes, 1973; Copeland and Antikarov, 2001).

1. The value of the underlying asset corresponds to lognormal distribution, or that returns are normally distributed. Changes in the value of the underlying asset are modeled as following a generalized Wiener process (geometric Brownian motion) where the expected return and variance are both constant.
2. There is only one source of uncertainty.
3. The underlying asset pays no dividends.
4. The option is contingent on a single underlying risky asset.
5. The option is European (i.e., it can only be exercised at maturity).
6. The volatility of the underlying asset is known and constant through time.
7. The exercise price is known and constant.

The basic idea behind the Black-Scholes formula is to create a portfolio consisting of the right number of units of the underlying risky asset, V, and a certain number of bonds, B, so that the portfolio has exactly the same payoff in each state of nature as the call option. They assume that it is possible to continuously adjust the portfolio in order to keep it risk-free. The risk of the portfolio is eliminated by continuously revising the option and the hedge ratio, a process called delta hedging. As the portfolio's value is independent of the change in the underlying asset value, such a risk-free hedged position should return the risk-free rate in equilibrium. The result of the model defines how the option value changes in terms of the change in the underlying asset value and time to expiration. The result is a complicated partial differential equation[13].

As will be shown later when discussing the binomial model, the basic idea behind the Black-Scholes model is the same as that of the binomial model. The only difference is the basis of deriving the Black-Scholes formula is from Ito Calculus (i.e., stochastic differential equations), while that of the binomial model is a simple algebraic approximation over discrete time interval. However, in the limit as the number of the discrete subintervals per unit of time becomes large, the multiplicative binomial model approaches a continuous solution like the Black-Scholes model (Copeland and Antikarov, 2001).

[13] Detailed mathematical presentation of the model is shown in Black and Scholes' (1973) article.

3.8.1.1 Limitations of Black-Scholes Model

The Black-Scholes model has been criticized due to a number of limitations. Most of these criticisms relate to the basic assumption underlying the derivation of the model. First, it assumes that the option can only be exercised at maturity (European options) because it only calculates the option value at one point in time, at maturity. This assumption eliminates the possibility of evaluating American options, which can be exercise at any time prior to maturity. Second, the model rules out the possibility of evaluating rainbow options since it assumes a single source of uncertainty. Also by assuming a single underlying risky asset, compound options are ruled out. But most investment decisions have compound options embedded in them because they occur in phases and there are usually several correlated sources of uncertainty. Other limitation of the Black-Scholes model relates to the sophisticated mathematical background required in order to properly understanding the derivation and the basic intuition of the model. This is because the model was derived from a variety of advanced mathematical techniques (e.g. Ito calculus), knowledge not common to corporate managers and practitioners (Graham and Harvey, 2000).

Although the Black-Scholes model is widely used for financial options valuation, the limitations make it very difficult to use for analyzing most real option (e.g., rainbow options, compound options). To be able to do that will require relaxing some of the underlying assumptions, which may significantly impact the results of the analysis. For instance in this pork case study, it is expected that management would exercise any of the real options available to the project at any time within the maturity period when market conditions are suitable. This requires that the options be evaluated as an American-type option. However, this is not possible when the Black-Scholes model is used because it calculates the value of the option at a point in time, the maturity date. Also the Black-Scholes model cannot be used to evaluate more than one option at a time. This means that the valuation of multiple options is ruled out when this model is used. With the hog investment having more than one option therefore, this model would not be suitable if the options are considered in combination. The next section presents a discussion on another real option valuation model that uses algebra and a lattice that is relatively easier to understand than a stochastic differential equation.

3.8.2 The Binomial Model

Cox et al. (1979) derived a pricing method based on the binomial approach and it has since been recognized as the simplest of the option pricing methods (Elton and Gruber, 1995). A detailed derivation of the model from Cox et al (1979) is presented in Appendix A[14]. They use probability theory to develop a binomial lattice approach to option pricing that employs discrete mathematics. The binomial option valuation model is based on a simple representation of the movement of the value of the underlying asset. With the binomial approach it is possible to price both European and American options. The model breaks down the time to expiration into a potentially very large number of time intervals or steps. A tree of the value of the underlying asset is initially produced working forward from the present to expiration. At each step it is assumed that the underlying asset will move up or down by an amount calculated using the volatility and

[14] The derivation extends the model to a multi-period valuation.

the time interval of the tree. Figure 3.1 presents an illustration of a two-step binomial tree of the path of the value of an asset, V.

From Figure 3.1, the initial value of the underlying, V, will move either up (by a multiplicative factor u) with probability p to V_u or down (also by a multiplicative factor d) with a complementary probability $1\text{-}p$ to V_d at the end of the first period. In the next period, the possible values are V_{uu}, V_{ud} or V_{dd}. The up *(u)* and down *(d)* movements are respectively estimated as (Cox et al., 1979):

$$u = e^{\sigma\sqrt{\Delta t}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\Delta t}} = 1/u \tag{3.2}$$

where Δt is the time interval of the binomial tree. The tree represents all the possible paths that the value of the asset could take during the life of the option. At the end of the tree, that is, at the expiration of the option, all the terminal option values for each of the final possible values are known since they are equal to their intrinsic values. The option values at each step of the tree are then calculated working back from expiration to the present. The option values at each step are used to derive the option value at the previous step of the tree.

The model assumes that the future values of the underlying asset follow a multiplicative binomial distribution over discrete periods[15]. With the multiplicative process, the values of the asset at the branches of the tree are bounded by zero at the bottom and approaches infinity as the number of period grows (Copeland and Antikarov, 2001). That is the probability distribution of the value of the asset approaches lognormal distribution as the number of periods increases. In addition, the model assumes the up and down parameters (u and d) and the volatility of the underlying asset (σ) are constant and known and uses risk neutral probabilities (p and $1-p$) as opposed to management's subjective probabilities for valuation. The binomial model employs two main approaches to valuing real options. These are the replicating portfolio approach and the risk neutral valuation approach.

3.8.2.1 Replication Portfolio Approach

The pricing of options is based on the construction of a portfolio[16] that consists of buying a particular number, m, shares of the underlying asset (also called the hedge ratio) and borrowing against them an amount, $\$B$, at risk free rate. It is assumed that the portfolio can be continuously adjusted in order to keep it risk free. The risk of the portfolio is eliminated by continuously revising the option contingent on the underlying asset and the hedge ratio, a process called delta hedging. Since the portfolio's value is independent of the change in the underlying asset value, such a risk-free hedged position should return the risk-free rate in equilibrium. Thus, the portfolio exactly replicates the future returns of the option contingent on the underlying asset in any state of nature. The equivalent portfolio would provide the same future returns as the option. In order to avoid risk free arbitrage profit therefore, they must sell for the same current price. As a result, an option can be value by determining the cost of constructing its equivalent portfolio. An example from Trigeorgis (1999, pg 72-77) is used to demonstrate the intuition of the replicating portfolio approach and how it is used to value options.

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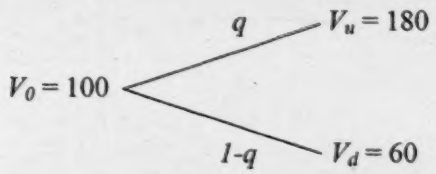
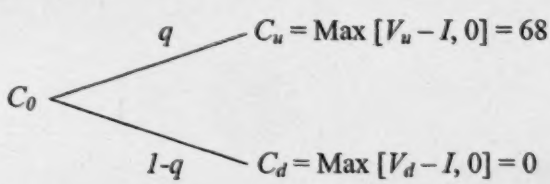
[15] Note that the value of the underlying asset could also follow as additive process but that will not be discussed in this thesis. An example of the additive process is presented in Copeland and Antikarov (2001).

[16] For a detailed discussion on the creation of replicating portfolio, interested readers are referred to Trigoergis (1999, pg 72) and Hull (1998, pg 233).

Assuming the initial value of the underlying asset, V_0 (currently worth \$100) will move over the next period either up to V_u = \$180 (i.e., with a multiplicative up parameter u = 1.8) or down to V_d = \$60 (with a multiplicative down parameter, d = 0.6) with probabilities q and $1\text{-}q$ respectively. This is illustrated in as follows:



Assuming the price of the underlying asset, I, is \$112 and the risk free interest rate, r, is 8%, then the value of the call option at the up state, C_u, and down state, C_d, at the end of the period is given as:



In the above illustration, the only unknown value is C_0. Now, assuming the replicating portfolio consists of m units of the underlying asset and $\$B$ of the risk free bond, the value of the portfolio at the end of the next period in the up and down state respectively are:

$$\begin{aligned} mV_u - B(1+r) &= C_u \\ mV_d - B(1+r) &= C_d \end{aligned} \tag{3.3}$$

Solving the two equations for m and B gives:

$$m = \frac{C_u - C_d}{V_u - V_d} = \frac{68-0}{180-60} = 0.56 \text{ shares} \tag{3.4}$$

and

$$B = \frac{mV_d - C_d}{1+r} = \frac{(0.56)60-0}{1+0.08} = \$31 \tag{3.5}$$

The number of units, m, of the underlying asset in the replication portfolio (equ. 3.4) is the incremental option payoff $(C_u - C_d)$ to the change in the value of the underlying asset $(V_u - V_d)$, also know as a hedge ratio or delta (Δ). The hedge ratio, m, multiplied by the value of the underlying risky asset at time zero, V_0, minus the value of the call option, C_0, gives a risk free payout, B_0 (eq. 3.6). Thus if one holds m units of the underlying asset and its value goes up, the capital gain would be offset by a capital loss in the short position created by the writing of the call (Copeland and Antikarov, 2001).

$$\begin{aligned} mV_0 - B_0 &= C_0 \\ mV_0 - C_0 &= B_0 \end{aligned} \tag{3.6}$$

Replacing the values of m and B_0 into equation 3.6 results in the following value for the call option;

$$C_0 = mV_0 - B_0 = 0.56(100) - 31 = \$25$$

Thus, with the replicating portfolio approach, the value of the flexibility from the simple example is given as $25. The same example is illustrated next with the risk-neutral probability approach.

3.8.2.2 Risk Neutral Probability Approach

An equivalent approach to the replicating portfolio method is the risk neutral probability approach. This approach is based on the construction of a hedge portfolio that consist of one share of the underlying risky asset and a short position in *m* units of the option that is being priced (e.g., a call option). The hedge ratio, *m*, is chosen such that the portfolio is risk free over the next short interval of time. If the hedge ratio is properly constructed the loss on the underlying asset is exactly offset by the gain on the short position in the call option and the result is risk free.

Using the same example as above, a risk free portfolio, ϕ, consisting of 1 share of the underlying risky asset and a short position in *m* shares of the options being priced can be created as follows;

$$\phi_0 = V_0 - mC_0 = 100 - mC_0$$

$$\phi_u = V_u - mC_u = 180 - m\,(68)$$

$$\phi_d = V_d - mC_d = 60 - m\,(0)$$

Now, the value of *m* must be chosen such that the portfolio is risk free. This is done by equating the end of period payoffs on the hedge portfolio. This is because if the value of the hedge ratio, *m*, that equates the end of period payoffs can be determined, the same cash flows can be generated by the portfolio in either state of nature and be risk free. Equating the two gives:

$$V_u - mC_u = V_d - mC_d$$

$$180 - m(68) = 60 - m(0) \qquad (3.7)$$

$$m = \frac{(u-d)V_0}{C_u - C_d} = \frac{(1.8-0.6)100}{68-0} = 1.764706$$

Given the value of the hedge ratio, *m*, the present value of the hedge portfolio is determined as:

$$V_0 - mC_0 = 100 - 1.764706C_0 \qquad (3.8)$$

Since the hedge portfolio is risk free, it should earn a risk free rate and the resulting payoff should be identical in either the up or down state. Multiplying the present value of the hedge portfolio by 1 plus the risk free rate and equating it to the payoff in the up state gives the following:

$$(V_0 - mC_0)(1+r) = V_u - mC_u \qquad (3.9)$$

Substituting the equation of the hedging ratio (eq. 3.7) into equation 3.9 and solving for the call value, C_0, gives the following result:

$$C_0 = \left[C_u\left(\frac{(1+r)-d}{u-d}\right) + C_d\left(\frac{u-(1+r)}{u-d}\right)\right] \Big/ (1+r) \qquad (3.10)$$

Equation 3.10 can further be simplified by defining

$$p \equiv \frac{(1+r)-d}{u-d} = \frac{(1+0.08)-0.6}{1.8-0.6} = 0.4 \qquad (3.11)$$

$$1-p \equiv \frac{u-(1+r)}{u-d} = \frac{1.8-(1+0.08)}{1.8-0.6} = 0.6 \qquad (3.12)$$

where p is a risk neutral probability, that is, the probability that will prevail in a risk neutral world where investors are indifferent to risk. The risk neutral probability depends on the magnitude of the up and down movements and risk-free interest rate. Inserting the risk neutral probability into equation 3.10, the value of the call option becomes:

$$C_0 = \frac{pC_u + (1-p)C_d}{(1+r)}$$

$$C_0 = \frac{0.4(68) + 0.6(0)}{1+0.08} = \$25 \qquad (3.13)$$

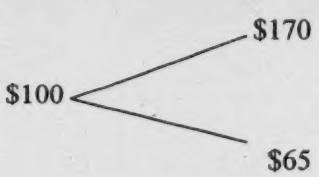
This is the same result obtained using the replication portfolio approach discussed earlier, which indicates that the risk neutral probabilities and replicating portfolio approach give the same results. In the next section, the risk neutral probability approach is used to evaluate the earlier example developed in chapter 2.

Example 3:

This example extends the real option analysis to evaluate the investment project in Chapter 2 (see Examples 1 & 2) with the risk neutral probability approach of the binomial model. It is an investment opportunity where an investment of \$115M has to be made in next year with certainty. Once the investment is made, the project has a 50-50 chance of generating either \$170M or \$65M. The risk adjusted discount rate for the investment is assumed to be 17.5% and the risk free rate is 8%.

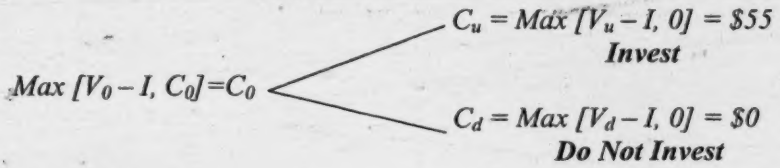
With this project, management can either commit to the project right now or defer the investment for one period before making a decision. Thus, the main real option embedded in the project is the option to defer. The managerial flexibility of being able to defer the beginning of the project gives management the chance to benefit from improvement in the market conditions. That is, management will not invest if revenue moves into the down state and this will save them the losses they would have incurred if they pre-commit. If revenue generation ends up in the upper state management will invest and benefit from the higher returns.

Using the risk-neutral approach, the option is valued as an American call. With the above information, the binomial tree for the project will look like as below:



where
Up movement *(u)* = 1.7
Down movement *(d)* = 0.65
Initial gross PV of the project *(V_0)* = \$100

The value of the option at the end nodes of the tree can then be determined as follows:



Where,
C_u = Call value at the up state = \$55
C_d = Call value at the down state = \$0

The risk neutral probabilities can be calculated using equations 3.11 and 3.12. as follows:

$$p \equiv \frac{(1+r)-d}{u-d} = \frac{(1+0.08)-0.65}{1.7-0.65} = 0.41$$

$$1-p \equiv \frac{u-(1+r)}{u-d} = \frac{1.7-(1+0.08)}{1.7-0.65} = 0.59$$

Using equation 3.13, the value of the American call option to defer the start of the project is given as:

$$C_0 = \frac{0.41(\$55)+0.59(0)}{1+0.08} = \frac{\$22.55}{1.08} = \$20.88$$

The option to defer the starting of the project is valued as \$20.88M. Adding this to the NPV with no flexibility value of -\$6.48M obtained in Chapter 2 gives an expanded project value with the flexibility to defer for one period to be \$14.40M. Thus when the option to defer was considered, the project now becomes economically viable. This means that the static NPV approach failed to capture the value of the managerial flexibility embedded in the project while the decision tree approach, although it accounted for the value of flexibility, used an incorrect discount rate[17]. Most importantly, the binomial approach used to value the above example did not make any additional assumptions or use any complex mathematics other than the same assumptions and algebra used for the static NPV analysis. Therefore if a manager is using an NPV to value a project without flexibility, then the manager has already made the necessary assumptions and has the required mathematical background to apply the real option methodology. The section that follows uses the concepts behind the one period scenario to extend the model to a multi-period horizon with discrete dividend adjustment.

[17] Copeland and Antikarov (2001) show that if the DTA uses corrected discount rates it gives the same value as the option approach.

3.8.2.3 Multi-Period Binomial Model With Dividends Adjustments

In this section, the risk neutral probability approach[18] of the binomial model is extended from the single period case described above to a multi-period horizon. The principles used in the single period example are the same for the multi-period framework. The example illustrated here considers a three-period horizon. In addition, this chapter also shows how the binomial model is adjusted to evaluate dividend-paying assets. Before the model is adjusted for dividends, the next section discusses some of the approaches used to adjust for the effects of dividends in the binomial model.

3.8.2.3.1 Types of Dividend Adjustment

There are several different ways in which dividends can be accounted for in the binomial model when valuing real options. Two of the different approaches to incorporating dividends in the binomial model are discussed in this section[19].

The first approach is where the underlying asset pays a continuous dividend yield (Kolb, 2000; Hull, 2002). In this case, a fixed annual dividend rate is determined and the rate is incorporated into the calculation of the risk neutral probability. With this approach therefore, the dividend forms a constant percentage of the value of the project. The adjustment implies that the probability of the underlying asset value increase varies inversely with the level of the continuous dividend rate (Kolb, 2000).

The second approach is where the asset is assumed to pay a discrete known dividend yield at a certain time period (Kolb, 2000; Hull, 2002). The known dividend yield, q, is assumed to be a proportion of the value of the underlying asset at the period in which the dividend is paid. When this approach is used, the dividend payments affect the value of the asset in the binomial tree and the loss of value from the asset will eventually affect the value of the option. In this case the binomial tree takes the form shown in figure 3.2. When dividend is accounted for in this manner, the holder of an American call or put option can decide to exercise the option early. In the case of an American call option, the holder may exercise the option immediately before the dividend payment (ex-dividend date) while the owner of an American put option may exercise the option immediately after the ex-dividend date. The is because the owner of the call may profit from the high asset value by exercising before the dividend payment while the put owner may benefit from the low asset value as result of the dividend payment. The process of valuing real option using the binomial model in a multi-period stage is discussed below.

3.8.2.3.2 Forward Calculation of the Binomial Tree

Figure 3.2 presents a three period multiplicative binomial process of the value of the project, V. The tree represents a forward calculation of all the possible values the asset could take in the future during the life of the option. Suppose in this illustration, the asset is expected to pay a known dividend yield which would be a proportion, q, of the asset value at time period $t = 2$, Figure 3.2 shows how the dividend is accounted for in the binomial tree[20].

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[18] Note that similar extension can be made using the replication portfolio approach.

[19] The reason why these two are being discussed is that they are the approaches used in the empirical model in chapter 5. For the remaining types of dividend adjustments, readers are referred to Kolb (2000), Hull (1998, 2002) and most other finance textbooks.

[20] Several known dividend yields during the life of the option can be dealt with in a similar fashion.

Starting at the first node at $t = 0$, the initial value of the project is V. At the next period, the value could move up to V_u which is calculated as V times the up ratio, u, or down to V_d with a value calculated as V times the down ratio, d. By following the same process, the value of the project in the next stage are V_{uu} (V times u squared), V_{ud} (V times u times d) and V_{dd} (V times d squared). However, because the asset is expected to pay a certain proportion of its value as dividends in this period, the effect of the dividend is to reduce the value of the asset as shown in Figure 3.2. This forward calculation of the value of the project in the binomial tree is repeated for every node until the values of the project at the end nodes of the tree are obtained. The values at the end nodes of the tree are the possible values of the project at the maturity of the options. In Figure 3.2 for example, the values at the end nodes are V_{uuu}, V_{uud}, V_{ddu} and V_{ddd}. Once all the possible values of the project have been determined in the binomial tree, the value of the options on the project can be calculated by working backwards from the end nodes (at expiration) of the tree to the present. From the above example, the values of a call option *(C)* and a put option *(P)* in the uppermost node (V_{uuu}) at expiration are respectively calculated as:

$$C_{uuu} = \max[V_{uuu} - I, 0] \text{ and } P_{uuu} = \max[I - V_{uuu}, 0] \qquad (3.14)$$

where I is the initial cost of the investment or the exercise price in option terminology. The process is repeated until all the option values at the end nodes are determined. In the case of a call, the option should be exercised if the value of the project is greater than the exercise price. On the other hand, put options are exercised if the exercise price is greater than the value of the project. The next step is to move back into the previous period ($t = 2$) to determine the decision management will take at that node. The value of call option at this node is calculated as follows:

$$C_{uu} = \max[e^{(-r\Delta t)}[pC_{uuu} + (1-p)C_{uud}], V_{uu} - I] \qquad (3.15)$$

where p is the risk neutral probability[21]. The first term of equation 3.15, $[e^{(-r\Delta t)}(pC_{uuu} + (1-p)C_{uud})]$, represents the present value of the option if it is kept alive while the second term, $V_{uu} - I$, is the value of the option if exercised. Thus, expression in 3.15 is interpreted to mean that management would choose between the value of the option if exercised or its present value if kept alive, whichever is larger. That is, if the present value of the option if kept alive is larger that the value of the option if exercised, the option would be kept alive and would not be exercised at this node. The option will be exercised only when its exercised value is greater than its value if kept alive. This valuation process continues working backwards through the rest of the nodes in the binomial tree. The value of the option to the project is obtained by the time the first node is reached at time $t = 0$.

3.8.2.4 Advantages and Disadvantages of Binomial Model

According to Trigeorgis (1999) and Copeland and Antikarov (2001), a major advantage of the binomial model is that, unlike the Black-Scholes model, it can be used to accurately price American options. This is because the binomial model makes it possible to check at every point in an option's life (at every step of the binomial tree) the

[21] Note that there is a slight difference between equation 3.15 and 3.10. However the formulas are identical except that the presentation in equation 3.15 is a continuous approximation while that in equation 3.10 is a discrete approximation. The continuous approximation is being used because as the step size increase, at the limit the binomial model approaches a continuous solution. The remainder of this thesis uses the continuous approximation.

possibility of early exercise. Second, the binomial model does not depend on the probability of certain outcomes. This means that the model is independent of investors that have different subjective probabilities about an upward and/or downward movement in the value of the underlying asset (Cox et al., 1979). As has been shown, the binomial model requires less mathematical background and skill to develop and use compared to the other option pricing models such as the Black-Scholes (Copeland and Antikarov, 2001). Also, the binomial model can be used to account for the effects of interactions among different options that may be present in a project. That is to say the binomial model can be adjusted for the valuation of multiple options.

On the other hand, one disadvantage is that, binomial models are trees and when used for modeling and valuing investment opportunities can quickly grow burdensomely large as the number of time periods increases. This is particularly true if the tree does not recombine (Copeland and Antikarov, 2001; Hull, 2002).

3.9 Chapter Summary

This chapter mainly described the methodology of the real option analysis. In order to comprehend the real options approach, however, a brief overview of financial options and the main similarities and limitations of the financial and real options analogy were discussed. Six different types of real options that could provide management with valuable operating flexibility and strategic adaptability were discussed. The chapter also shows that most of the input variables required for real options analysis are determined by using the NPV without flexibility model.

The two most commonly used real option valuation models were discussed. First, the Black-Scholes option-pricing formula was described and it was shown that the intuition behind its derivation is similar to that of the binomial model except that the Black-Scholes model uses stochastic differential equations while the binomial model uses a simple algebra and lattice. Also the underlying assumptions of the Black-Scholes formula make it difficult for most real option applications. The binomial approach, on the other hand, is based on a representation of the movement of the value of the underlying asset over time. The chapter showed how a binomial lattice could be used to value options on the underlying asset that follows a multiplicative stochastic process. If the binomial lattice is multiplicative, then the model approaches the Black-Scholes formula as the number of subintervals per year becomes large. Two equivalent approaches to valuing options using the binomial approach are the replicating portfolio approach and the risk neutral approach. Both approaches were used to value a simple call option and showed that if properly applied they give the same results. It is important to note that the same intuition and procedure used to value the call option in example 3 can be extended to the valuation of other options like put and compound options.

The chapter extends the one period scenario to a multi-period case and also shows the binomial model can be adjusted to account for dividend payments. A dividend-paying asset enhances the value of a put option but decreases the value of a call option. Without dividends however, the owner of an American option may not exercise until the maturity date. Finally, the example of a simple project evaluated in chapter two was re-evaluated with the real options approach. The NPV approach ignored the value of managerial flexibility. Although the DTA did consider flexibility, it is flawed because it assumes a constant discount rate when clearly uncertainty does not stay the same at the various nodes of the tree. The real options approach accounts for the value of flexibility by

forming a replicating portfolio and as a result eliminating the possibility of theoretical arbitrate profits. However, the possibility of forming a replication portfolio for capital projects that are not traded is one of the shortcomings of the real option approach.

3.10 Tables For Chapter Three

Table 3.1: Terminal Payoffs of Options Position

Option Position	Payoff
Long Call	Max [S – X, 0]
Long Put	Max [X – S, 0]
Short Call	-Max [S – X, 0]
Short Put	-Max [X – S, 0]

Source: Hull, 2002, pg 164.

Table 3.2: Determinants and their Effects on Option Values

	American		European	
Increase in Factor	**Call Value**	**Put Value**	**Call Value**	**Put Value**
Stock Price	+	-	+	-
Exercise Price	-	+	-	+
Time to Maturity	+	+	?	?
Risk-free Interest Rate	+	-	+	-
Volatility	+	+	+	+
Dividend	-	+	-	+

Source: Hull, 2002, pg 183.

Table 3.3: Impact of the Various Variables on Each Model's Outcome

Variables	Static NPV	Call Option Value
Increase in uncertainty (σ)	↓	↑
Increase of risk free rate (r)	↓	↑
Increase in time to defer (t)	↓	↑

Source: Trigeorgis (1999), pg 373

Table 3.4: Comparison of Variables Use to Value Financial and Real Options

Financial Options	Real Options
Current value of stock *(S)*	Present Value of expected cash flows *(V)*
Exercise price *(X)*	Investment cost *(I)*
Time to expiration *(T)*	Time until opportunity disappears *(T)*
Stock value uncertainty (σ)	Project value uncertainty (σ)
Risk free interest rate *(r)*	Risk free interest rate *(r)*
Dividend *(d)*	Free cash flows* paid out by the underlying asset *(q)*

Source: Trigeorgis (1999), pg 125.

*Free cash flows are defined as the per period cash inflows less its corresponding cash outflows, excluding the initial cost of the investment that leave the business.

Table 3.5: Annual Income and Expenditure Statement

Annual Revenue		$1200
Annual Variable Cost	-$500	
Annual Fixed Cost	-$200	-$700
Annual Net Cash flows		$500

3.11 *Figures for Chapter 3*

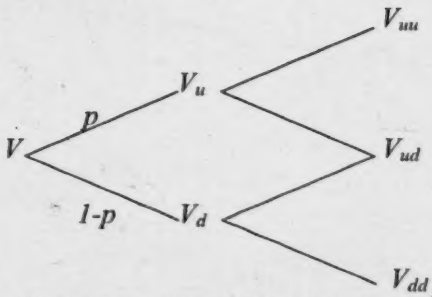

Figure 3.1: A Two Step Binomial Tree of the Value of the Underlying Asset

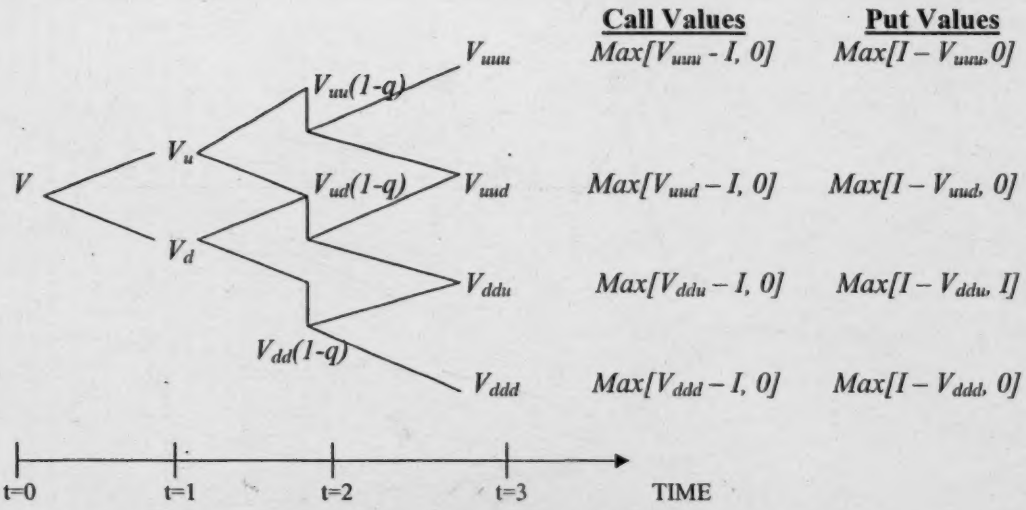

Figure 3.2: Forward Calculation of Movement of Project Value with Discrete Dividend Adjustment

CHAPTER 4: THE PORK CASE STUDY AND REAL OPTIONS IDENTIFICATION

In order to apply the real options valuation to investments in hog production in Western Canada, it is important to have some background knowledge of the hog case study being used as well as the Canadian hog industry. This chapter gives a brief overview of the production trend of the Canadian and Western Canadian[22] hog industry. The main sources of risk facing hog producers are discussed. Also presented in this chapter are the description of the case study for this thesis, the data and the type of real options that may be valuable for the project. The methodology used to value the real options in the pork case study is explained.

4.1 Overview of the Canadian Hog Industry

The Canadian hog industry has grown significantly in recent years as shown by the increase in hog population. Figure 4.1 depicts the trend of the number of pigs in Canadian and Western Canadian farms from 1981 to 2001. The number of hogs produced in Canada was 9.86 million head in 1981 and has increased steadily since then to 13.96 million head in 2001 (Census of Agriculture, 2001), an increase of about 41% over the period. The national situation is not different from what is happening in the Western part of the country. The Western Canadian hog industry has also shown similar growth over the same period. The number of pigs produced has doubled over the period from 2.9 million head in 1981 to 5.84 million in 2001. Quebec and Ontario are the leading producers of pigs in Canada with 30.6% and 24.7% of total Canadian production respectively in 2001 (Census of Agriculture, 2001). Manitoba leads in the production of pigs in Western Canada with 43% of the total pigs produced in Western Canada in 2001 followed by Alberta (35%), Saskatchewan (19%) and BC (3%). The increasing trend of the number of hogs produced continued into 2002. Canadian hog producers reported 14.7 million head on farms at October 1, 2002, an increase of 2.3% over the previous year. The national average growth over the past five years has been 5% per year (Statistics Canada, 2002).

Along with the increase in production, the size of pig farms in Canada and Western Canada has also grown. The change in scale of production is reflected by the average number of pigs reported per farm in Statistics Canada census reports from 1981 to 2001 (Figure 4.2). The average number of pigs reported per farm in Western Canada is higher than that of Canada in general for the entire period. However, pig farms are getting fewer in number over the same period (Figure 4.3). The number of pig farms in Canada has decreased by over 72% from 55,765 in 1981 to 15,476 in 2001 while that of Western Canada has also decreased by about the same margin from 26,617 in 1981 to 7,142 in 2001.

The number of hogs slaughtered in Canada is also showing an increasing trend. Slaughtered hogs in the third quarter of 2002 were 5.5 million, up 7.8% from 2001 (Saskatchewan Agriculture, 2002). During the first nine months of 2002, the national slaughter reached a record level of 16.3 million hogs, 6.6% higher than for the same

[22] Western Canada comprises the provinces of Manitoba, Alberta, Saskatchewan and British Columbia (B.C).

period last year. Similarly, the number of hogs slaughtered in Western Canada increased by 9.5% over the same period (Statistics Canada, 2002).

According to Alberta Agriculture Food and Rural development (2001), one of the reasons that have contributed to the increased pig production is that today's hogs grow faster and sows produce more piglets than they did in 1981. Also the increased international demand, particularly from the United States and Japan as well as new markets such as Mexico has also contributed to the increased production (Statistics Canada, 2001). A number of tariff reductions, combined with the relatively low value of the Canadian dollar compared to the United States dollar and plentiful supplies of feed grain, have made Canadian pork increasingly attractive to international markets. In January 2000, countervailing duties on pigs exported to the United States were dropped for the first time in almost 15 years (Statistics Canada, 2003). Increased demand from U.S. finishing operations boosted the export of live feeder hogs by 57% in the five years ending in December 2000, while increasing demand for pork from the United States and Japan put meat exports 76.3% higher in 2000 than in 1996. Exports of live hogs to the United States rose by 5.3% over the third quarter of 2001. In 2001, exports soared by 23% over the previous year (Saskatchewan Agriculture, 2002).

The increased pork production notwithstanding, the price of hogs has been variable. Canadian average hog prices in the third quarter of 2002 were about 28% lower than during the same period in 2001 (Statistics Canada, 2002). According to Statistics Canada (2002), the decline of the Canadian hog prices has been attributed to the performance of the U.S. hog markets. American hog price has been declining since the beginning of 2002 and this has affected the slaughter price on the Canadian market. Falling prices in the United States has been attributed to the slowdown in the disposition of pork supplies and a higher slaughter volume (Statistics Canada, 2002).

There are several types of hog operations in Canada. Examples include farrow to finish, farrow to wean, grower to finish, among others. The type of operation that is evaluated in this study is the farrow to finish. Farrow to finish operation involves keeping male and female breeding stock together in order to produce finishing pigs for the market. Most commercial farrow to finish operations in Canada takes place within a year round controlled environment buildings (Manitoba Agriculture and Food, 2002). These buildings are specifically designed and equipped for the gestation, farrowing, nursery, growing and finishing stages of producing market hogs. As a result producers are able to farrow their pigs throughout the year thereby marketing pigs all year round. The advantage of this system of hog production is that it significantly reduces productivity losses associated with stress of movement, adaptation to new environments and transmission of diseases (Manitoba Agriculture and Food, 2002). In addition, producers are able to monitor the performance of animals through to maturity, thereby observing final results of breeding techniques and other management practices.

Many different breeding techniques are available to hog producers but artificial insemination is becoming the most popular method of breeding compared to using boars to inseminate the herd (Manitoba Agriculture and Food, 2002). However, many farrow to finish operators keep boars in the operation for cleanup breeding or to tease females in order to decrease time to breeding. The sow to boar ratio of farrow to finish farms will thus vary according to the breeding program. Replacement females are either purchased

from the open market or operators resort to in-house replacements where the gilts are selected from within the herd.

Hog production in Canada and Western Canada is getting increasingly intensive (Alberta Agriculture, 2001). Producing hogs on a large scale requires a significant amount of initial capital to start. Producers usually resort to borrowing from financial institutions. It is therefore important that hog producers identify the major sources of risk in pork production and deal with those risks in order to remain profitable. This is particularity important if the investment is irreversible, as may be the situation in farrow to finish hog production. Failure to do so may result in significant losses with the consequences of not being able to meet the financial responsibility of borrowers. Following is a discussion on the main sources of risk faced by hog producers in Canada.

4.1.1 Sources of Risk in Pork Production

Risk is defined as the chance of an uncertain outcome, particularly exposure to unfavorable outcome (Hardarker et al, 1997). This definition implies that risk focuses mainly on the bad things that may or may not happen. However, when risk is measured by standard deviation it could be defined as the likelihood of deviation from expected values or forecast. This definition encompasses both upside and downside risks. Generally, there are five broad categories of risk in agriculture (Hardarker et al, 1997). These are production risk, price or market risk, institutional risk, human or personal risk and financial risk. However, the main sources of risk to hog producers are production and price or market risks (Bauer, 1988).

Production risk is associated with the variability in the quantity and quality of the animal being produced. This type of risk occurs because hog production is usually affected by many uncontrollable events that are often related to diseases, mortality and morbidity rates, genetic issues, and so on. Technology could also play a key role in production risk. For instance, the introduction of new production techniques often offer the potential for improved efficiency, but may at times yield poor results, particularly in the short term. Producers usually manage production risk through sound production practices, frequent animal vaccination to prevent diseases and technological improvements in production (Brealey and Meyers, 1991).

Price or market risk reflects risks that are associated with unpredicted changes in the price of output or inputs that may occur after the commitment to production has been made. Input and output prices affect producers from different angles. This is because they have limited or no control over the prices they pay and the prices they receive. This type of risk may come about as result of increased competition, changes in supply and demand conditions, among others. Hog production, for instance, typically requires up-front investments in feed and equipment that may not produce returns for several months or years. Because markets are generally complex and involve both domestic and international considerations, producer returns may be affected by unpredictable events both domestically and in other regions of the world. One of the techniques producers can use to manage price risk is by engaging in investment strategies such as contract production, future markets, forward contracting and diversification into other activities or investments (Brealey et al, 1991; Unterschultz et al., 1998)

4.2 Project Description

The case study to be analyzed is a farrow to finish pork operation. It is the case of a business that has recently invested in a 2600 farrow-to-finish pork operation in Western Canada[23]. The business supplied a detailed weekly and monthly production projections and other investment information. Investment costs, input and output data necessary for this analysis were extracted from the projections. The general structure of the 2600 farrow to finish project is presented in Figure 4.4. The project is situated on a very large tract of land that makes it possible for on-site manure disposal. Before the purchase of the land, there is the need for feasibility studies such as water testing, regulatory siting approval etc. to be carried out. This is necessary not only to ensure that the land is suitable for hog production but also to obtain the necessary building permits for the construction of the barns. Once the land is purchased, the project starts with the building of the farrowing and dry sow barns. This is followed by building separate weaner barns on the same site. By design, there would be no onsite grower and finisher barns for this project. At post-weaning, the piglets would be contracted to a number of private firms or individuals who will be responsible for housing the animals until they are ready to be sold. One reason for this arrangement is to help management control production risks such as the spread of disease. The arrangement enables management to practice all-in and all-out grower-finishing barns that allows a break between batches of finishing pigs for disease control.

For this particular project, the farrowing and the dry sow building unit is expected to be completed within five months after building construction begins. Immediately after the building is completed, the breeding animals would be purchased and housed in the finished building while the weaner barns are being constructed. Breeding animals are expected to be purchased in a total of five batches. The first batch would be brought into the barn on the 17th week after the beginning of the project. This batch will constitute about 23% of the total sows to be purchased. The same quantity of breeding animals would be purchased in the second, third and forth batches on the 20th, 24th and 28th weeks respectively. The last batch of the breeding animals will come in on the 32nd week and they would constitute the remaining 8% of the total sows. At post-weaning, the piglets would be contracted to a number of private firms or individuals who will be responsible for housing the animals until they are ready to be sold. There would be no pig sale until the 61st week. All pigs would be sold at the finishing stage, which implies there would be no sale at the weaner or grower stages. Production and sale of animals would not reach a steady state until at the beginning of year three. A steady state is reached when the unit is in full production all year round and remains constant afterwards into perpetuity. Based on these projections, the budget for the 2600 farrow-to-finish pork operation is presented below.

4.2.1 Budget for the Operation

The budget for the 2600 farrow to finish pork project is categorized into three main parts. The first part consists of the initial capital cost of the investment, the second

[23] For confidentiality reasons, certain details of the business and its data will not be discussed.

is revenue generation and the third is the cost of production. Details of each category of the budget are described below[24].

The initial capital cost of the investment is subdivided into four categories for the project. These are the cost of land, building and equipment costs, cost of production inventory (breeding stock) and operating cost. Table 4.1 presents the breakdown of the initial capital for the 2600 hog project. Because the business contracts out the piglets at post weaning, the capital expenditure on building and equipment for grower and finishing barns is not included in this budget. However, this business is expected to pay for all the variable cost incurred from growing to finishing during the period of the contract. Nevertheless, the cost of building and equipment represents the largest group of capital requirements for the hog project. The costs involved in the construction of the operational site are derived from a variety of sources. Some of the facilities in the building include feed bins, ingredient bins, breeding and gestation, water supply, manure wagon, site preparation, electrical, fire alarm system and so on. The cost of building and equipment is estimated at $2730 per sow. For 2600 sows, the building and equipment costs is approximately $7.1 million, constituting about 73% of the total initial cost of the investment. The cost of breeding sows during the start up phase is calculated to be $300 per sow, about $150 premium over market price of finished animals, while the cost of a boar is $1500 each. The sow to boar ratio is 90 to 1, which means about 28 boars are required for the 2600 sows. The total cost of production inventory (sows and boars) is estimated as $822,000. The assumed annual rate of sows and boars replacement for the analysis is 45% and 50% respectively when full unit production is reached. The farrow to finish operation occupies a tract of land that cost the enterprise $1.3 million and the operating cost (working capital) of the project is assumed to be $0.5 million. Thus, the total capital cost for the 2600 farrow-to-finish operation is estimated to be $9.72 million.

Table 4.2 depicts the breakdown of the annual revenue generation for the hog project at steady state of production. The main source of revenue generation to the pork operation is the sale of finished hogs, representing about 98% of the total revenue generation of the project. Finished hogs are sold at an average dressed weight of 89.49kg and each sow produces an average of 22.61[25] head per year for a total of 58,786 hogs marketed annually. At a market price of about $149 per pig sold, the total revenue from the sale of finished hogs is estimated to be $8.8 million per annum or about $3,380 per sow when the unit is in full production. The other source of revenue for the pig project is the sale of culled animals (sows and boars) which constitutes the remaining 2%. Each year, 45% of the sows and 50% of the boars are culled and sold. With a 2% cull sow death loss, the number of sows culled every year is 1,197 at an average weight of 200kg each. On the other hand, 14 boars are culled every year at a weight of 225kg per animal. At a sale price of $146 and $170 for each sow and boar culled respectively, the total sales from culled animals contribute about $177,000 per year to the hog enterprise. In total, the annual revenue for the 2600 farrow-to-finish unit is approximately $8.98 million, which is about $3,450 per sow.

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[24] Note that the budget for revenue generation and cost of production are estimated at steady state of production only.

[25] The figure is net of all death losses. Death losses were assumed to be as follows: 8% for Pre-weaning, 1.5% for post weaning and 3% for grower/finishing.

The annual cost of production for the project consists of feed costs, cost of labour, offsite and management costs, other variable and fixed costs, and annual purchase of replacement sows and boars. A breakdown of the annual expenditure of the hog project at steady state is depicted in Table 4.3. Feed cost is the major production input to the pork production process, representing about 60% of the total annual expenditure. Feed barley is assumed to constitute 65% of the total feed cost with wheat, corn, soybean meals and others making the remaining 35%. The budget for feed cost covers the feed of the animals from birth until they are sold, as well as the feed for the breeding sows and boars. From birth to market, pigs are fed at a cost of $0.83 per kg dressed, which is about $74 per pig marketed. The total feed cost per year is estimated to be $4.35 million. Annual labour cost represents about 7.7% of the total annual expenditure of the pork operation. The budget assumes an annual total labour expense to include manager's salary and benefits, salaries and benefits of full-time and part-time employees, payroll taxes, and any paid family labour. The annual cost of labour is $9.60 per pig marketed. Thus, the total annual labour cost comes to approximately $565,500. The offsite and management costs constitute the cost of contracting out the grower and finishing pigs to a private enterprise. The private enterprise will be responsible for housing the animals until they are ready to go to the market. The offsite and management cost for the enterprise is estimated as $21 per pig marketed. This brings the annual offsite and management costs to $1.24 million, which is about 16.9% of the total annual expenditure. Expenditure in the category of the other variable cost include bedding, genetic and veterinarian expense, medical supplies, repairs and maintenance, transportation and marketing expense, utilities, cost of artificial insemination, etc. The annual additional variable costs for this farrow-to-finish hog operation is calculated as $11.73 per pig sold. The fixed cost consists of property tax, insurance, miscellaneous barn expenses, rental expenses, office and administrative expense, etc. The fixed cost per year for this analysis is calculated as $2.13 per pig marketed. The total fixed and other variable costs are estimated as $815,000 per year (about 11% of the total annual expenditure). Annual replacement sow and boar purchase constitutes the lowest expenditure (about 4.9% of the total annual expenditure) to the pig production process. These animals are purchased to replace the 45% and 50% of the sows and boars that are culled every year in order to keep production at full capacity all year round. A total of 1,222 sows and 14 boars are purchased annually as a replacement at a cost of $275 per sow and $1,500 per boar. This costs approximately $357,000 per year[26]. The total annual production cost is estimated to be $7.32 million when the unit is in full production.

4.3 Possible Real Options in the Hog Project

There are various types of real options that could exist theoretically (i.e., see section 3.4). However, some of these are likely to have limited or no impact on the valuation of a farrow to finish investment. Which options are valuable depends on the specific characteristics of the project and in which industry it is present. It is therefore important to identify the options that would be most valuable for the hog project.

[26] Note that it is possible to use in-house multiplicative replacement at a cheaper cost but this analysis assumes that commercial replacement is used.

4.3.1 Option to defer

The option to defer is potentially valuable in projects that have to cope with economic uncertainty and irreversibility. In such cases, a waiting strategy could increase the value of the project. Both factors are very prominent in investments in farrow to finish projects. Hog producers have limited or no control over the prices they receive and pay for their outputs and inputs respectively. In addition, investing in hog production is partly irreversible because the cost of constructing the barn may not be fully recovered. Used barns may have little or no resale value. This is because if a business is not doing well due to a fall in output prices or an increase in input prices, then the rest of the industry may be experiencing similar problems and may therefore not be interested in buying additional barns. The factors described above make this option attractive to the farrow to finish project. However, investment in pork production is not monopolistic and therefore it is expected that the effects of competitive entry may impact the ability of management to defer the beginning of the project. Trigeorgis (1991; 1999) and Smit and Ankum (1993) show that the effects of competition can be treated similarly to known dividends on American call options. The effect of this is to increase the possibility of early exercise of the option to defer. By incorporating known dividend yield in the analysis therefore, it is assumed that the impact of competition is being accounted for since it increases the possibility of early exercise by decreasing the value of the underlying asset and the defer option contingent on the asset value.

4.3.2 Compound Sequential Option (Staged Investment)

This option is valuable when the initial cost of the investment of the project is incurred in stages. This gives management the chance to abandon the project at some stage of the investment process if the investment becomes economically unattractive. The investment in the hog project can be looked at in two stages. The first stage investment involves the purchase of the land after feasibility studies and the second stage investment involves building the barn and purchasing the breeding animals. If after the first stage investment management decides not to continue with the second stage investment, the project can be abandoned and the land sold for it salvage value. Thus, this option to abandon the project in midstream during the development stage and not invest further if new unfavourable information is received could be valuable for this project.

4.3.3 Option to Alter Operating Scale

This option involves the managerial flexibility of either expanding or contracting (production capacity) or temporary shutting down and restarting operations. The option to contract production capacity would not be valuable for the hog project due to the low salvage value likely associated with this investment. Similarly, the option to temporarily shut down operation is not likely to be valuable for this project as well. This is because of the nature of the biological process in pork production. Pigs born today take several months to complete the weaning, growing and finishing cycles of the production process. This implies that management will continue to feed the animals when operation is temporary shut down until they complete the cycle. The only option in this category that is considered for the hog project is the flexibility of expanding the production capacity of the project due to improved market conditions and/or better production than originally

anticipated. This gives management the chance to benefit from the improved market and/or production conditions and as a result increase the profitability of the project.

4.3.4 Option to Abandon

The option to abandon the project could be valuable if the project has a reasonable salvage value. For a farrow to finish project, this will depend on the ability to sell the land, breeding stock and the hog barn. Hog barns are generally not easy to sell and may be sold at a huge discount. Thus, hog investments may not command a reasonable salvage value to make this option attractive to this kind of investment. However, if unfavourable market conditions persist for a long time and there are not enough signs of market improvement, this option could be exercised to save the operation from further losses. Thus, the option to abandon is considered for this project.

4.3.5 Growth option

This option could be of a significant value if management intend to launch a follow-on project in the future. In order words, there could be a growth option to launch a new project if the initial project is successful. Since the business did not indicate any intentions of making a follow-on investment in the near future, this option is not considered for this project.

From the above discussion, the options that may be more valuable for the hog operation are the option to defer due the irreversibility and economic uncertainties associated with the project, the option to expand in order to take advantage of improved conditions and the compound sequential option which gives management the chance to abandon the project after each investment stage. The option to abandon could also be valuable for this project but the value is largely dependent on what can be realized in terms of salvage value. These are the four options that are evaluated for the farrow to finish project in this study. The methodology used to evaluate the option is discussed next.

4.4 Calculating the Pork Case Study Real Options

This section shows how the four different real options identified for the 2600 farrow to finish hog project are calculated. As indicated earlier, the binomial option-pricing model is used in the evaluation of the options identified for the hog project. One of the reasons that motivated the use of the binomial model for the hog project is the possibility of determining the value of more than one option in combination. Also, this model makes it possible to evaluate the options in the pork case study as American-type options. The implication is that at any time within the maturity date of the options, management can exercise any of the options when market conditions are suitable. Assuming the binomial tree for the hog project is the same as in Figure 3.2 in chapter 3, the values of each of the options are evaluated as follows

4.4.1 Calculation of the Option to Defer

This option gives management the flexibility to defer undertaking the hog project and therefore allows management to benefit from favourable random movements in the project value, whilst at the same time they cannot be hurt by unfavourable market circumstances. This is because they do not have the symmetric obligation to invest. Therefore, management could defer the beginning of the project and invest later (any

time within the maturity date of the option) when the option value within that period turns out to be favourable. In other words, the option to delay the pork project is evaluated as an American call option on the project value, V, with an exercise price equal to the required initial capital cost, I. This translates into the right to choose the maximum of the project value minus the required investment or zero, since management will simply not invest if the project value does not cover the necessary costs. The required initial cost for this project comes from the costs of the land, barn construction, breeding animals and operating cost. The payoff for the option to defer at the uppermost end node of the binomial tree in Figure 3.2 is expressed as:

$$C_{uuuu} = \max[V_{uuuu} - I, 0] \tag{4.1}$$

That is to say management will invest only when the value of the project at that node is larger than the initial investment cost. After determining the value of the option at the end nodes, equation 3.15 in chapter 3 is used to calculate the value of the defer option to the hog project. That is working backwards through the nodes of the tree to determine at each node if the option should be exercised or kept alive. This calculation is repeated at each node of the tree working backwards until the value of the option is obtained at the first node *(t = 0)*. The option to defer is valuable to the hog project if its value at present (at the first node) is greater than zero.

4.4.2 Calculating the Option to Expand

If market conditions turn out to be more favourable than initially anticipated, management can expand the scale of production by incurring an additional capital cost. This is analogous to an American call option to acquire an additional part of a base scale project by paying the additional cost as the exercise price. For this project, it is assumed that management can expand the size of the project by 50% by incurring an additional cost, I', if market conditions are favourable than initially expected. This means that any time within the maturity period of the option, management has the flexibility to either maintain the same scale of operation and receive project value, V_{uuu}, at no extra cost, or to expand the scale of the operation by 50% and receive a 50% increase in the project value by paying the additional cost, whichever is higher. For this hog project, the cost of expansion would result from building addition barns, purchasing more breeding animals and operating cost. The payoff of this option at the end node of the tree is expressed as:

$$C_{uuu} = \max[0.5V_{uuu} - I', 0] \tag{4.2}$$

This means that management will exercise the option to expand at this node if the net benefit of the expansion *$(0.5V_{uuu} - I')$* is greater that zero. This calculation is repeated until all the values at the end nodes are obtained. Stepping back to the previous period to determine whether the option should be exercise at that node or kept alive is expressed as follows:

$$C_{uu} = \max[e^{(-r\Delta t)}(pC_{uuu} + (1-p)C_{uud}), 0.5V_{uu} - I'] \tag{4.3}$$

Thus, if the value of the option to expand when exercised is greater than the value if kept alive, then management will exercise the option if the value of the project ends up in that node else the option would be kept alive. The option to expand is valuable to this project if the value at present *(t = 0)* is greater than zero.

4.4.3 Calculating the Option to Abandon

The pork operation could be abandoned in exchange for its salvage value if market conditions turn out to be unfavourable. The salvage value is realized from the sale of the project at the time of abandonment. The option to abandon the pork operations is valued as an American put option with an exercise price equal to the project's salvage (abandonment) value, I_A. This option translates into the flexibility of choosing the maximum of the project's abandonment value, I_A, minus the present value of the project in its present use, V_{uuu}, or zero. The payoff for the option to abandon at the end nodes of the tree is given as:

$$P_{uuu} = \max[I_A - V_{uuu}, 0] \tag{4.4}$$

If I_A is greater than V_{uuu}, then management would abandon the project at that node. On the other hand, if V_{uuu} is greater than I_A, then the project should continue. The calculation is repeated until all the values at the end nodes are obtained. The option value is worked backwards from the end nodes to the previous period using the risk neutral probability to determine whether the option should be exercised or kept alive. This is calculated as:

$$P_{uuu} = \max[e^{(-r\Delta t)}(pP_{uuu} + (1-p)P_{uud}), I_A - V_{uu}] \tag{4.5}$$

The backward calculation continues until the option value is obtained at the present node *(t = 0)*. Once again, the option to abandon is valuable to the pork project if its value is greater than zero.

4.4.4 Calculating the Multiple Options

In order to capture the effects of the interactions among the options for this project, the value of the options has to be calculated in combination since their combined value may differ from the sum of their separate values. According to Trigeorgis (1993a), the presence of subsequent options increases the effective underlying asset for prior options and as a result exercise of a prior real option may alter the underlying asset and hence the value of subsequent options on it, causing an interaction. The degree of interaction is however related to the option type and the degree of overlap of exercise region (Trigeorgis, 1993a).

Consequently, the combined value of the option to abandon and expand is calculated for this project by following Copeland and Antikarov (2001). The combined value of the options to expand and abandon is evaluated by treating the options as a mutually exclusive decision at each node, and the decision that results in the highest payoff is chosen as optimal. This is because at any point in time, only one of the two options could be exercised by the management of the hog project. That is, the hog project cannot be abandoned and expand at the same time. By treating the options as mutually exclusive at each node ensures that the valuation of the multiple options meets this condition. The payoff structure for the multiple options is given as:

$$CP_{uuu} = \max[I_A - V_{uuu}, 0.5V_{uuu} - I^{'}, 0] \tag{4.6}$$

Equation 4.6 implies that at each node of the binomial tree, management would either abandon, expand or leave the project in its current state depending on which option presents the highest value. The process is repeated until all the values at the end nodes of the tree are obtained. The next step is to work backwards to determine whether the

project should be expanded, abandoned or the options should be kept alive. This is calculated as follows:

$$CP_{uu} = \max[e^{(-r\Delta t)}(pCP_{uuu} + (1-p)CP_{uud}), I_A - V_{uu}, 0.5V_{uu} - I'] \qquad (4.7)$$

The value of the multiple option is obtained by working through all the nodes in the binomial tree to the present *(t = 0)*.

The option to wait was not valued in combination with the options to abandon and expand using this approach. This approach requires that the options being valued together should not be dependent. However, the option to defer and the other two options being considered here are not independent and as a result this approach would not be appropriate to use for that purpose.

4.4.5 Calculating the Sequential Compound Option

In order to evaluate the sequential compound options for the hog project, the investment in the project is split into two phases. The first phase investment is the purchase of the land within a certain time frame after the feasibility studies. The feasibility studies are necessary for getting the required environmental approval (e.g., regulatory siting, adequate water supplies, etc.) for the project to go ahead. Since management has the right but not the obligation to purchase the land, this can be looked at as a call option with an exercise price equal to the cost of the land. The second stage investment involves the building of the barn and the purchasing of the breeding animals within a certain time frame with an exercise price equal to the cost involved. The second stage investment is contingent on the first stage investment being curried out. This is because if the land is not purchased, the barn cannot be built and the project cannot proceed. By staging the investment, it allows management to decide on whether to abandon the project after the first stage or continue with the project by making the second stage investment. If management decides to abandon the project after the first stage, the land can be sold for its salvage value. At the end of the first period (i.e., at the maturity date), the first option expires and therefore it must be exercised at the cost of the land or left unexercised at no cost. If exercised, the payoff is not directly dependent on the value of the underlying project but on the value given by the option to invest at the second stage. In valuing the stage investment option therefore, the value of the option to build the barn (second stage) at present *(t = 0)* is determined first. The value of the option to purchase the land (first stage) is then determined based on the tree of the option to invest in the second stage and not the value of the underlying asset. This is because to determine whether the land should be purchased or not depends on the value of the final project which also depends on the second stage investment. The payoffs at the end nodes of the binomial tree for the option to invest in the second stage are determined as follows:

$$C_{uuu} = \max[V_{uuu} - I'', 0] \qquad (4.8)$$

where I'' is the cost of constructing the barn and purchasing the breeding animals. Equation 4.8 implies that management will invest if the value of the project is greater than the cost. Having determined all the payoffs at the end nodes, the value of the option to go ahead with the second stage investment is calculated using the backward calculation technique. That is, stepping back into the previous node the decision to either exercise or keep the option alive is determined until the value of the option is obtained at time $t = 0$.

Having determined the option to invest in the second phase, the value of the option to invest in the first stage, which is the value of the compound sequential option, is

determined based on the decision tree provided by the option to invest in the second stage. Thus, the end node payoff for the option to invest in the first stage is expressed as follow:

$$C'_{uuu} = \max[C_{uuu} - I_L, 0] \qquad (4.9)$$

where I_L is the cost of the land and C_{uuu} is the corresponding value from the tree of the option to invest in the second stage. Working backwards through the tree, the value of the option if exercised and its value if kept alive are determined at each node of the tree based on which management would make the optimal decision. If management decides to exercise the first option by paying the exercise price, the first option uncovers the second option immediately. The value of the option to invest in the first stage, which is the value of the compound sequential option, is determined at time period $t = 0$. According to Copeland and Antikarov (2001) the value of the compound sequential option can be interpreted as the present value of the project today given that there are two investment phases. As a result the project should be rejected if the cost to be incurred (if any) before the first stage investment is greater than the value of the compound option, otherwise the project should be accepted. Since the hog project requires feasibility studies on the land before it is purchased, the value of the compound sequential option can be interpreted as the amount management should pay for the cost of the feasibility studies. As a result, if the cost of the feasibility studies is greater than this value, the project should be rejected. For this hog project, the time to maturity for the first stage and second stage investments for the compound sequential option is set at one and half and five years respectively. The choice of the time to maturity is discussed further in chapter 5.

4.5 Tables for Chapter 4

Table 4.1: Breakdown of Initial Capital Cost for 2600 Farrow-to-Finish Hog Unit

Items	Description	Price ($/Unit)	Total
Land			$ 1,300,000
Building and Equipt. Cost	2600 Sows	$ 2731/Sow	$ 7,100,000
Production Inventory:	2600 Sows	$ 300/ Sow	$ 780,000
	28 Boars	$ 1500/ Boar	$ 42,000
Operating Capital			$ 500,000
	Total Initial Capital Cost		$ 9,722,000

Table 4.2: Breakdown of Annual Revenue for 2600 Farrow-to-Finish Unit at Steady State of Production

Items	Description	Price ($/Unit)	Total
Marketed Hogs	22 head/sow @ 89kgs each	$ 150/animal	$ 8,800,000
Cull Sows	@ 200kgs each	$ 146/ animal	$ 175,000
Cull Boars	@ 225kg	$170/ animal	$ 2,380
		Total Annual Revenue	$8,977,380

Table 4.3: Breakdown of Annual Expenditure for 2600 Farrow-to-Finish Unit at steady State of Production

Items	Price ($/Unit)	Total	Per Sow
Feed Cost	$74/ pig sold	$ 4,350,000	$ 1,673
Labour	$9.62/pig sold	$ 565,500	$ 217
Offsite Costs	$21/ pig sold	$ 1,236,000	$ 475
Other Variable Costs	$12/ pig sold	$ 689,600	$ 265
Fixed Costs	$2.13/ pig sold	$ 125,000	$ 48
Replacement Sows	$ 275/ animal	$ 336,000	$ 129
Replacement Boars	$ 1500 / animal	$ 21,000	$ 8
Total Annual Expenditure		$ 7,323,100	$ 2,816

4.6 *Figures for Chapter 4*

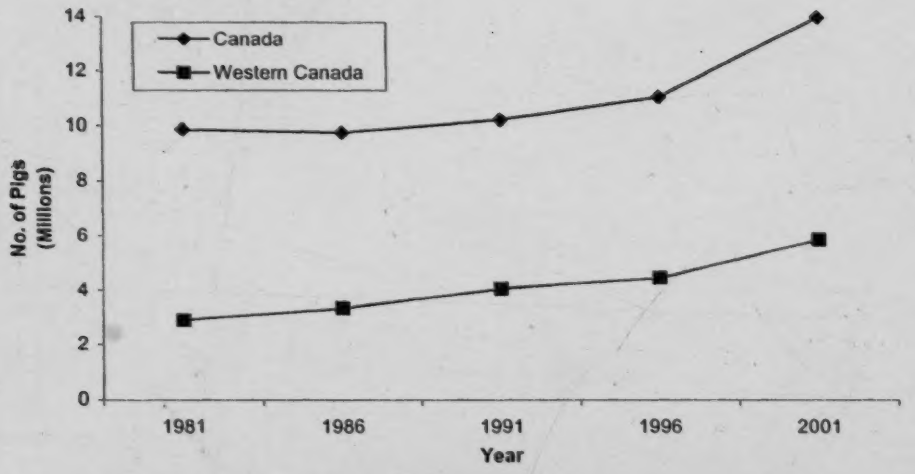

Figure 4.1: Number of Pigs Produced in Canada and Western Canada (1981-2001)
Source: Statistics Canada (2002)

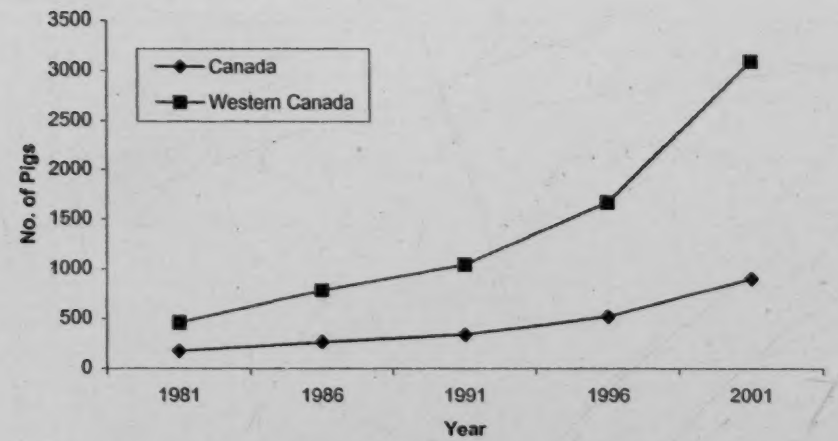

Figure 4.2: Number of Pigs Per Farm in Canada and Western Canada (1981-2001)
Source: Statistics Canada (2002)

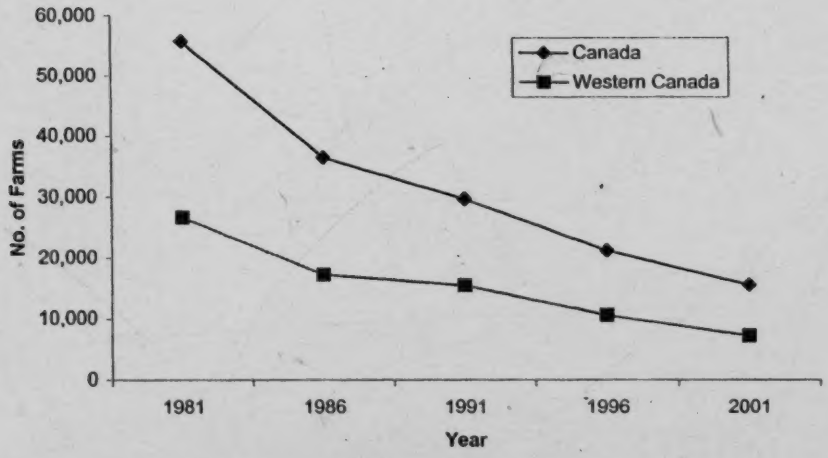

Figure 4.3: Number of Pig Farms in Canada and Western Canada (1981-2001)

Source: Statistics Canada (2002)

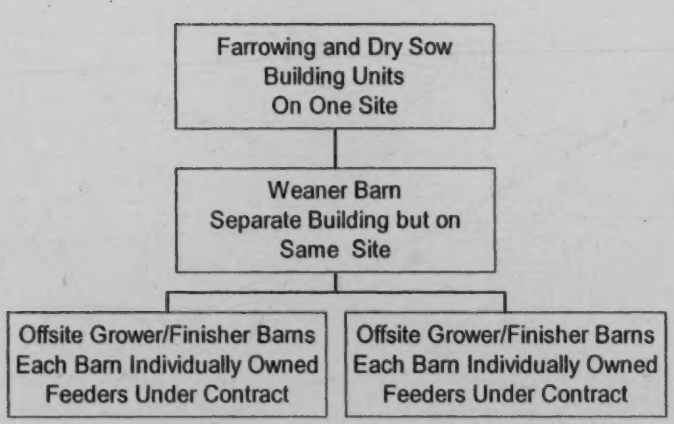
Farrowing and Dry Sow Building Units On One Site
Weaner Barn Separate Building but on Same Site
Offsite Grower/Finisher Barns Each Barn Individually Owned Feeders Under Contract
Offsite Grower/Finisher Barns Each Barn Individually Owned Feeders Under Contract

Figure 4.4: The General Structure of the 2600 Farrow to Finish Pork Project

Source: Case Study

CHAPTER 5: ANALYSIS AND RESULTS

This chapter presents the quantitative valuation and results of the 2600 farrow-to-finish hog investment project. The results of the analysis presented in this chapter are based on the data received from the case study and the real options identified in Chapter 4. The valuation starts with the calculation of the NPV without flexibility. This is followed by the valuation of the separate values of the real options identified for the hog project. The combined value of the abandon and expand options is calculated next. Lastly, the ENPV of the project, which is the value of the project with managerial flexibility, is calculated. Sensitivity analysis of the results is presented in the next chapter.

5.1 Static NPV Calculation and Results

The NPV without flexibility for the 2600 farrow to finish hog project is calculated using the traditional technique described in chapter 2.1. The model used is the risk adjusted discount rate approach (equation 2.1). To calculate the NPV, first the gross present value of the hog project is determined, followed by the initial investment cost. The initial investment cost is then subtracted from the gross present value to get the NPV of the project. The following assumptions were made for the calculation of the NPV.

1. A risk-adjusted discount rate of 15% was chosen for the analysis. The reasoning for the choice of the discount rate relates to the general risk characteristic of hog investments. According to Trigeorgis (1999), high-risk investments usually have discount rates of above 20% while low risk investments are usually discounted with rates of below 10%. In between are investments in the medium risk category which have discount rates in the region of 10% - 20%. Given the main sources of risk associated with pork investments (see section 4.1), it was assumed that this project will fall within the medium risk category and hence a 15% discount rate was chosen. Relating this discount rate to the discussion in section 2.1.2 and in particular the CML model (equation 2.3), the rate of risk premium for this hog project is 11% given the current risk free rate of 4%. This compensates management for bearing the risk associated with the hog project.
2. It was also assumed that the project would run to perpetuity. This is because the investors of the pork case study intend to carry on with the project for a long time. As a result the calculation of the cash flow projections for the project was made for the first 15 years after which a perpetuity calculation was added to capture the value of the project beyond the 15 years[27]. The static NPV calculation has constant free cash flows after the second year when the project reaches a steady state production. This is because input and output prices are constant over time and production is constant at steady state. As a result the output and input prices used for estimating the value of the project in perpetuity were the steady state values for the 15-year period.
3. It is also assume that the kind of dividend policy adopted does not affect the value of the project and therefore does not alter the NPV of the project. Copeland and Weston (1983) show that in a world without taxes, it makes no difference whether

[27] The reason why the NPV was estimated up to 15 years before capturing the perpetuity value is because the case study provided cash flow projections for 15 years. It was therefore decided that the analysis will use of all the cash flow information provided by the case study after which the perpetuity value of the project would be captured.

shareholders or investors receive their cash flows as dividends or as capital gains. Therefore dividend policy does not affect the value of a firm if there are no taxes, agency cost or information asymmetry. Since the NPV analysis of the hog project does not include income tax, different dividend policies should not affect the NPV of the project.

The first step in the NPV calculation is to determine the gross present value of the project, V_0. This is calculated using equation 5.1 below.

$$V_0 = \sum_{t=0}^{15} \frac{E(c_t)}{(1+k)^t} + \frac{AE(c)/k}{(1+k)^{15}} \qquad (5.1)$$

where $E(c_t)$ is the expected free cash flows, k is the risk adjusted discount rate, t is the time period and $AE(c)$ is the project's expected free cash flows for perpetuity. The first term of the equation calculates the value of the project for the first 15 years while the second term captures the perpetuity value.

The gross present value is calculated by subtracting the cost of production per period from the revenues generated per period to get the free cash flows (FCF) per period. The FCF in each period are then discounted with the risk-adjusted discount rate of 15% to get the discounted FCF for each period. The sum of the discounted FCF for all the periods gives the gross present value of the hog project. Table 5.1 presents how the gross present value for the 2600 farrow to finish project is calculated. For this project, the gross present value, V_0, is estimated to be $7.1 million.

The second step is the calculation of the initial cost of the investment, I. The initial investment cost for the hog project comprises the cost of land, construction cost, cost of breeding animal and operating capital (see Table 4.1). The initial cost of the investment for the project is expected to be incurred within the first 2 years of the beginning of the project. As a result, future investments were discounted to present. The present value of the initial cost of the investment is calculated using equation 5.2 below:

$$I = \sum_{t=0}^{2} \frac{I_t}{(1+k)^t} \qquad (5.2)$$

where I_t is the investment made in period t. A larger percentage of the cost (about 85%) is expected to be incurred at the beginning and the first period whilst the remaining 15% would be incurred in the following period. To capture the full effect of the discounting factor, the initial investment cost is discounted on monthly basis for this analysis. The expected future investments were discounted with the 15% risk-adjusted discount rate. The risk-adjusted discount rate is used because it is assumed in this project that the future capital expenditures are not certain. The sum of the discounted investment gives the present value of the initial cost of the investment. Based on the information provided by the case study, the expected initial cost of the investment, I, is estimated to be $8.59 million.

The static NPV of the hog project is calculated by subtracting the present value of the initial investment cost from the gross present value of the project. For the 2600 farrow to finish project, the NPV was estimated to be $-1.49 million, which suggest that the project is not worthwhile to undertake. Base on the NPV without flexibility analysis therefore, the project would be rejected because it has a negative net present value. Valuing the project with the presence of managerial flexibility could however make the investment economically attractive. Before the identified real options are evaluated

however, the next two sections discuss the calculation of the discrete dividend yields and the volatility of the project value used in the real options analysis.

5.2 Calculation of Discrete Dividends

The rate of dividend accounts for the 'leakage' in the value of the underlying asset. This represents the loss of value of the underlying asset as result of the free cash flows paid out of the business to the owners or due to competitive entry. As indicated in chapter 3 (section 3.8.2), there are several different ways in which dividends can be accounted for in real options valuation but this thesis adopts two of the approaches.

The first is the continuous dividend yield approach where a fixed annual dividend rate is determined and the rate is incorporated into the calculation of the risk neutral probability (see section 5.4.3 below). This approach assumes that the asset pays continuous dividends even in periods when free cash flows are negative. That is, the dividend paid out in each time period of the binomial tree is approximately $q^*V_{(j)}^*\Delta t$, where $V_{(j)}$ is the value of the project in time period j, Δt is the time interval of the binomial tree and q is the continuous dividend rate. The annual dividend rate for the continuous adjustment is set at 5% for this hog project. The choice of this rate was guided by the calculation of the discrete dividends from the NPV model of the hog project. The average of the discrete dividends during the life of the option was approximately 5.26% and this is presented below.

The second approach is where the asset is assumed to pay a discrete known dividend yield annually. In this case the annual dividend yields during the life of the option are calculated on a per period basis depending on the free cash flows generated by the project in that particular period in which the dividend would be paid. Thus, the dividend yield may vary from period to period based on the free cash flows in that period. Following Copeland and Antikarov (2001, pg 252) the NPV without flexibility model is used to calculate the annual dividend yield for the hog project. The calculation procedure requires that a certain percentage of the FCF be paid out every year. Based on the information received from the case study, it was assumed that 50% of the positive FCF is paid out to investors annually. This is in accordance with the dividend policy of the management of the hog project to pay 50% of the free cash flows from the project to the investors. With this approach the project pays no dividend in periods in which negative free cash flows are generated. The present values of the dividends per year are calculated by multiplying the 50% by the present value of the FCF per period. The next step is to calculate the present value of the project with dividends and this is done by summing the PV of the FCF starting from the year in which the present value is being estimated for to perpetuity. In year 3 for example, the PV of the project with dividends is estimated by summing the PV of the FCF from year 3 to perpetuity. The dividend yield per year is calculated by dividing the PV of dividends by the PV of the project value with dividends. Table 5.2 presents the estimated annual dividend yields for the hog project during the life of the options[28]. The binomial tree is adjusted to account for the discrete dividend by following the discussion in Copeland and Antikarov (2001 pg. 252) and Hull (2002, pg 363) and this is illustrated in Figure 3.2 in Chapter 3.

[28] For detailed discussion on this approach and how the dividend yield are calculated and incorporated into the binomial model, readers are referred to Kolb (2000, pg 478), Copeland and Antikarov (2001, pg 252) and Hull (2002, pg 363).

5.3 Estimating Volatility

The standard deviation of returns (volatility), σ, is one of the most difficult variables to estimate in real option valuation. The volatility for the hog investment is estimated using the Monte Carlo simulation approach in Copeland and Antikarov (2001) discussed in section 3.7.1 of this thesis. This approach incorporates the main sources of risk of a project into a spreadsheet and use Monte Carlo simulation to estimate the standard deviation of the rate of returns based on the distribution of the present values.

In estimating the volatility, it was assumed that the farrow to finish project has two main sources of risk: production risk and market or price risk. Production risk is associated with the variability of the number of finished hogs marketed per year. This risk results from the unpredicted changes in factors that influence production such as diseases, mortality rate, genetic issues, among others. On the other hand, price risk is associated with the variability of returns to the hog operation due to unpredicted changes in output and/or input prices. Consequently, three variables were modeled as being stochastic to reflect the above sources of risk associated with the pork operation. These are: (1) the price of finished hogs to reflect output price risk, (2) price of barley to reflect input price risk and (3) the number of hogs sold per sow per year to reflect production risk. The price of barley is used to reflect input price risk because according to Manitoba Agriculture and Food (2002b) barley is the main feed ration in pork production in Canada, constituting about 65% of the total feed cost.

Following the identification of the sources of risk, the prices of hog and barley were modeled as following a mean reverting stochastic process. A presentation of a mean reversion stochastic process is in Appendix B. The decision to model the prices as mean reverting is because theory and research (e.g., Pindyck and Rubinfeld, 1991; Dixit and Pindyck, 1994; Schwartz, 1997) have shown some evidence of commodity prices being mean reverting. Following Schwartz (1997) and Dixit and Pindyck (1994), the price of finished hogs and barley were estimated by running the following AR(1) regressions:

$$X_{t,i} = \beta_{0,i} + \beta_{1,i} X_{t-1,i} + \varepsilon_{t,i} \qquad (5.3)$$

where $X_{t,i}$ is the price at time t for product i, $X_{t-1,i}$ is the price at time t for product i lagged one period, $\beta_{0,i}$ and $\beta_{1,i}$ are the parameters to be estimated, i is hog or barley, and $\varepsilon_{t,i} \sim N(0,\sigma)$ is independent and identically distributed *(iid)* random error which serves as the source of price risk.

Using Alberta monthly data of hog and barley prices from August 1991 to February 2002 obtained from Alberta Agriculture, a seemingly unrelated regression (SUR) was estimated using SHAZAM software 8.0 to determine the parameters in equation 5.3 for hog and barley prices. The data was deflated to 2002 prices using the Canadian consumer price index for all items from CANSIM. For the price process to be mean reverting $\beta_{1,i}$ must be greater than negative one but less than one ($-1 < \beta_{1,i} < 1$). The estimated parameters are reported in Table 5.3. The goodness of fit of the model is tested using the R-square. The R-square of 0.9537 indicates that about 95% of the variation in the prices of hog and barley in this month are explained by their respective prices of the previous month. The estimated parameters for both hog and barley prices were significant at 1% and also within –1 and 1. This is an indication that both prices exhibit mean reverting characteristics. To determine whether the prices of hog and barley

are correlated, a test for contemporaneous correlation is performed with a null hypothesis of no correlation. The test was done by comparing the calculated likelihood ratio from SUR to the critical Chi-square (χ^2) value with 1 degree of freedom from the χ^2 table (Griffiths et al., 1993). The critical χ^2 value with 1 degree of freedom from the table at 1% level of significance is 6.64 less than the observed value of 24.14. The test therefore rejects the null hypothesis and suggests that contemporaneous correlation between the error terms of hog price and barley price exist. This also indicates that the use of SUR for the estimation of the parameters in the model is appropriate. The correlation coefficient between the error terms of hog and barley prices was estimated to be 0.21.

The parameters in Table 5.3 were used to predict the monthly future prices of hog and barley using equation 5.3. In predicting the future prices, the @Risk simulation software in Microsoft Excel was used to draw the random errors. The random errors are defined for each variable using the standard error from the regression analysis. Because the price of hogs is correlated with the price of barley over time, the effects of correlation was accounted for by adjusting the error term of one of the equations, barley in this case, using equation 5.4 below (Hull, 1997 pg 361):

$$\varepsilon_b^* = \rho_{hb}\varepsilon_h + \varepsilon_b\sqrt{1-\rho_{hb}^2} \qquad (5.4)$$

where ε_b^* is the adjusted error term for the price of barley, ρ_{hb} is the correlation between the error terms of hog and barley prices, ε_h and ε_b are the independently drawn random errors for hog and barley prices respectively from @Risk normal distribution. Having generated the random errors, the price at time period t is forecasted by using the price at $t-1$ as the independent variable. The forecasted price of this month is therefore a component of the price of last month, the parameters estimated from the SUR and a random error generated from @Risk.

Figures 5.1 and 5.2 depict the predicted future monthly stochastic prices for hog and barley respectively. The figures illustrate one iteration of a simulation of a possible path of prices through time. To compare mean reversion characteristics to that of random walk, a 90% confidence interval estimate of the upper and lower values of the predicted hog and barley prices were generated from @Risk. A random walk model was also used to predict hog and barley prices and a 90% confidence interval of the upper and lower values generated from @Risk[29]. The random walk model uses the same constants from the SUR results but β_1 is equated to 1. Equation 5.3 is then used to generate the random walk prices. Figures 5.3 and 5.4 depict a comparison of the predicted mean reverting and random walk prices for hog and barley at 90% confidence interval respectively. Both figures show that the prices predicted by the mean reversion model are closer to their mean over time than the random walk prices. This indicates that with the AR(1) model, the mean reversion process is less likely to forecast negative prices than the random walk model. The predicted stochastic monthly prices of hog and barley were then aggregated to yearly prices by finding the average value in a twelve-month period.

Production risk was incorporated into the analysis using @Risk triangle distribution with the respective best and worse case scenarios of 26 and 18 pigs marketed

[29] Note that the predicted random walk prices for both hog and barley were not used in the Monte Carlo analysis.

per sow per year, net of all death loss. The figures for the best and the worse case scenarios were selected based on the production information provided by the case study. The mean of 22 pigs marketed per sow per year is also based on the expected pig production per sow per year figure from the case study. With the selected best and worse case scenario figures, the expected number of pigs marketed per sow per year generated from @Risk corresponds to the value used for the static NPV analysis.

Following Copeland and Antikarov (2001), the stochastic variables, i.e., the forecasted prices of barley and hog, and production, were then incorporated into the NPV model in Excel to calculate the standard deviation of the rate of returns of the gross present value of the hog investment. The formula for calculating the rate of returns is given as:

$$\frac{V_1}{V_0} - 1 \tag{5.5}$$

where V_0 and V_1 are determined respectively as:

$$V_0 = \sum_{t=0}^{15} \frac{E(c_t)}{(1+k)^t} + \frac{AE(c)/k}{(1+k)^{15}} \quad \text{and} \quad V_1 = \sum_{t=1}^{15} \frac{c_t}{(1+k)^{t-1}} + \frac{c/k}{(1+k)^{14}} \tag{5.6}$$

where c_t is the free cash flows for period t and c is the free cash flow for perpetuity. The remainder of the variables in V_0 and V_1 are the same as previously defined in equation 5.1. V_0 is the present value of the expected free cash flows calculated at the expected values of prices and production (i.e., from the static NPV). V_1 is the discounted expected free cash flows from year 1 calculated with the simulated prices and production generated from Monte Carlo. The value of the rate of return from equation 5.5 was simulated in Monte Carlo on 10,000 iterations to generate its distribution and standard deviation. The standard deviation is used in the real option analysis as the volatility of the gross present value of the hog project.

The simulation results indicate that the rate of return is symmetric as depicted in Figure 5.5. This is consistent with the lognormal assumption implied by the multiplicative binomial process of the gross project value. The annual standard deviation of the distribution of returns, which is the annual volatility for the hog project from the Monte Carlo simulation, is calculated to be approximately 43.5%. The high volatility implies that investment in pork production is fairly risky.

As indicated earlier, the volatility of the project returns is a very important component of the real option analysis. Real option values are quite sensitive to the volatility of the project returns (Copeland and Antikarov, 2001). The estimated volatility of the hog returns indicates that investments in hog projects are risky. The high volatility of returns obtained for the hog project is not surprising given that the predicted hog and barley prices from the Monte Carlo simulation analysis were volatile, particularly the price of hogs (see Figure 5.1 & 5.2). In a study conducted by Bresee (1997), a historical simulation analysis estimated the volatility of Alberta hog returns to be 54%, which is higher than the figure from the Monte Carlo analysis.

Using a Canadian data from the TSE 300 from 1948 – 1997, Ross et al. (1999, pg 249-255) determined the long term Canadian market risk premium $(E[R_M] - R_f)$ to be between 4%-7% and the standard deviation of the Canadian market (σ_m) to be between 16% - 18%. With a risk adjusted project discount $(E[R_p])$ rate of 15% and a risk free rate (R_f) of 4%, the volatility of the project (σ_p) implied by the CML should be between 36% -

45%. The volatility of 43.5% given by the Monte Carlo simulation analysis falls within the range implied by the CML when a 15% discount rate is used.

Several reasons could account for different volatility estimates. Copeland and Antikarov (2001) contend that fixed costs have a positive influence on the volatility of project returns. As a result a project with a large percentage of fixed cost is likely to have a higher project value volatility than a similar project with a lower percentage of fixed cost, all things being equal. Davis (1998) also indicates that the volatility of a project value can be influenced by the initial gross present value of the project. A study by Marceniuk (2000) at University of Saskatchewan (U of S) determined the volatility of returns for hog production in Saskatchewan to be about 19%. However, the results from the U of S study may be low. This is because Unterschultz et al (1998) and Maung and Foster (2002) all found the volatility of hog price alone to be around 20%. Since the volatility of the hog project returns results from the build up of the many uncertainties (e.g., hog price, feed cost and production) that are associated with the project, it is expected that the volatility of the project returns should be higher than just the volatility of price. For instance, Davis (1998) indicates that volatility of a project value should be greater than or at least equal to the volatility of its price if there is no operating flexibility. However, if the project incurs fixed costs then the volatility of the project returns should be higher than the volatility of the price, all things being equal. Following this argument therefore, the volatility of hog project value should generally be higher than that of its price since hog production generally incurs fixed costs. From the above discussion therefore, the estimated 43.5% volatility for this analysis from the Monte Carlo is acceptable.

5.4 Real Option Calculation and Results

Four types of real options are evaluated for the 2600 farrow-to-finish hog investment project using the risk neutral probability approach of the binomial option-pricing model described in chapter 3.8.2. The types of real options as discussed previously in chapter 4 are:

1. The option to defer the timing of the project.
2. The option to expand the production capacity if market conditions improve.
3. The option to abandon the project if market conditions deteriorate.
4. The sequential compound option.

For this hog project therefore, management has the right but not the obligation to proceed with the investment project. Therefore, the timing of the investment can be deferred for a certain period of time in order to benefit from future favourable developments. Should management proceed to invest and subsequent market conditions are better than initially anticipated management can expand the size of the operation so as to benefit from such improvements. On the other hand, if market conditions deteriorate severely, management can abandon the project some time in the future. The sequential compound option determines the maximum amount management should pay for feasibility studies and also allows management to decide whether to abandon the project after the first phase investment or continue by investing in the second phase.

To assess the impact of different dividend adjustments and variable exercise prices on the values of the real options, four different models were estimated for the first three options. One of the reasons why dividend is being incorporated into the option

model is that the pork case study pays a proportion of the cash flows generated by the project to the its investors. Since dividends could play a role in the decision of management to exercise an option or not, it was necessary that the impact of dividends be accounted for in the real option analysis. The second reason is to capture the impact of competitive entry on the option values and for that matter management decision-making, particularly the option to defer. This is because pork production in Canada is carried out in a competitive environment and therefore it is expected that competition could affect the value of the pork project and options contingent on the project.

The models estimated for the first three options are:

1. Model 1 uses a continuous dividend adjustment with constant exercise prices. Kolb (2000) and Hull (2002) indicate this approach as one way of incorporating dividends in the binomial model when valuing options.
2. Model 2 adopts the continuous dividend adjustment approach of model 1 but assumes that the exercise prices of the options increase at the risk free interest rate over time[30]. Luehrman (1998) indicates that business opportunities do not usually have constant capital expenditure. Some of the factors that may account for variable capital expenditure include interest rates, inflation among others. Luehrman (1998) therefore suggested that one of the extensions that could be made to model 1 is to vary the exercise price over time. Model 2 explores this extension.
3. Model 3 uses the discrete dividend adjustment approach with constant exercise prices for all the options. Hull (2002) and Copeland and Antikarov (2001) also indicate this approach as one way of adjusting for the effects of dividends on option values when using the binomial model. Given the way management of the hog project deals with cash payouts, the discrete dividend adjustment may be a better presentation for this case study.
4. Model 4 adopts the discrete dividend adjustment approach of model 3 with the exercise prices increasing at the risk free rate over time.

Before the options are evaluated, the additional necessary input variables for the binomial model were determined. Table 5.4 shows the calculated input variables used for the valuation of the first three real options for the pork project. The variables were determined as follows:

5.4.1 Gross Present Value and Initial Investment Cost

The gross present value *(V_0)* of the hog project is the present value of the expected cash flows from the project, excluding the initial cost of the investment. This was determined from the traditional NPV without flexibility model (see Table 5.1). The estimated V_0 for this project is \$7.1 million. Similarly, the initial cost of the investment *(I)* is calculated using the NPV without flexibility framework. For the 2600 farrow to finish hog project, I is calculated to be \$8.59 million.

[30] Note that increasing the exercise price at the risk free rate may not be the best approach. Luehrman (1998) indicates that one way to look at this is to determine the probability distribution of the exercise prices and use the uncertainty of the distribution to vary the exercise price over time. However, the purpose here is just to determine the impact of the increasing strike prices on the option values.

5.4.2 *Time to Maturity*

The time to maturity *(T)* for a real project is the time left until the option disappears. For a project like farrow to finish hog project, *T* is generally not as pre-defined as for a financial option. For the first three options, it is assumed that the opportunity for management to exercise the options would exist for five years. This implies that the time to maturity for the options is set at five years. Having set the time to maturity variable, there is also the need to determine the step size or the time interval *(Δt)* of the tree. The time interval determines the number of times the value of the project moves in a year. According to Copeland and Antikarov (2001), the accuracy of the option value improves as the number of times the binomial tree moves in a period increases. Depending on the life of the option, usually Δt of between 0.1 and 0.5 years is very common in real option applications that use the binomial model. For this analysis therefore, the time interval *(Δt)* of the binomial tree is set at 0.2. This implies that the value of the project will move five times within a year in the binomial tree. In the case of the compound sequential option, it is assumed that the opportunity to invest in the first and second stages would exist for one and half and five years respectively and Δt for this option was set at 0.25.

5.4.3 *Other Model Parameters*

The risk free interest rate used for this analysis is 4%. This is the rate for a 5 year GIC from Canadian Imperial Bank of Commerce (CIBC) as at February 10, 2003 (CIBC, 2003).

The up and down ratios were calculated using equation 3.2 in chapter 3 as follows:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.435\sqrt{0.2}} = 1.21$$

$$d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.435\sqrt{0.2}} = 0.83$$

Similarly, the risk neutral probability with and without dividend adjustment are respectively calculated as follows (Hull, 2002):

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} = \frac{e^{(0.04-0.05)0.2} - 0.83}{1.21 - 0.83} = 0.45$$

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{(0.04)0.2} - 0.83}{1.21 - 0.83} = 0.47$$

5.4.4 *Movements of the Project Value in the Binomial Tree*

Using the up and down ratios, the forward movement of the gross present value of the hog project in the binomial tree are calculated in Microsoft Excel by following the example in Figure 3.2. Since the maturity date of the options is 5 years and the value of the project will move 5 times in a year, the binomial tree for the value of the project will have 25 steps during the life of the options. Starting at the first node at $t = 0$, the value of the project is \$7.1M. At the next step of the tree, the value of the project could move up to \$8.59M (i.e., the initial value of \$7.1M multiplied by the up ratio of 1.21) or down to \$5.89M (i.e., \$7.1M multiplied by the down ratio of 0.83). This process is repeated until the values of the project at the end nodes of the tree are obtained. The forward calculation of the gross present value of the hog project in the binomial tree without dividend

adjustment is presented in Figure C1 in Appendix C[31]. Having determined the input variables and calculated the movements of the value of the project in the binomial tree, the next step is to evaluate the real options in the project. To demonstrate how the option values are calculated, only the continuous dividend adjustment with constant exercise price (model 1) is used. The other three models follow similar processes.

5.4.5 Option to Defer

The option to defer the timing of the hog project is valued analogous to an American call option, with an exercise price equal to the initial investment cost. This gives management the right but not the obligation to invest in the hog project at a price of $8.59 million any time during the next five years. The backward calculation of the value of the option to defer the hog project is presented in Figure C2 in Appendix C. The value of this option is determined by first calculating the intrinsic values of the option at the end nodes of the binomial tree in Figure C2. If the intrinsic value at any particular end node is greater than zero, management will invest if the value of the project ends up in that node, else management will not invest. Starting at the uppermost end node which has a value of $919.24M, the intrinsic value for the option to defer the timing of the hog project in that node is calculated using equation 4.1 as follows:

$$\text{Max}\,[\$919.24\text{M} - \$8.59\text{M}, 0] = \$910.65\text{M}$$

Thus, the intrinsic value of the defer option at the uppermost end node is $910.65 which means that management will invest if the value of the hog project ends up in that node within the next five years. On the other hand, the intrinsic value of the option at the down end node of the tree which has a value of $0.05M is also calculated as follows:

$$\text{Max}\,[\$0.05\text{M} - \$8.59\text{M}, 0] = \$0$$

This also implies that management will not invest if the value of the project ends up in that node within the next five years. This process is repeated until all the values at the end nodes of the tree are obtained. Once the values at the end nodes are determined, the next step is to move back in time to period $t = 24$ to determine whether the option should be exercised or kept alive in the nodes in that period. This is done using equation 3.15. Again using the uppermost value at $t = 24$, the value of the option if exercised is calculated as:

$$\$756.73\text{M} - 8.59\text{M} = \$748.14\text{M}$$

On the other hand, the value of the option if kept alive is calculated by using the intrinsic values of the uppermost and second uppermost nodes in period $t=25$. This is calculated as follows:

$$= e^{(-0.04*0.2)}[0.45*\$910.65M + 0.55*\$614.36M] = \$741.73M$$

The decision at this node is made by comparing the two values and choosing whichever is larger as follows:

$$\text{Max}\,[\$748.14\text{M}, \$741.73\text{M}] = \$748.14\text{M}$$

From the above calculations, the value of the option if exercise is greater than the value of the option if kept alive at this node. It would therefore be optimal for management to exercise the option (i.e., invest in the hog project) if the value of the project ends up in that node. The values of the option for the rest of the nodes in this time period are calculated using the same procedure and comparing the values. The process

[31] Appendix C discussed in more detail how the binomial tree of the value of the hog project is constructed. It also discusses the backward calculation of the real options using the defer option as an example.

continues working backwards for the rest of the nodes. The value of the option to defer the hog project is obtained by the time the first node is reached at time $t = 0$. The calculation of the option to defer for the other three models follows the same steps. The results of the real options analysis for each of the models for the hog project are presented in Table 5.5.

With the continuous dividend adjustment and fixed initial cost of investment (model 1), the option to defer the beginning of the hog project is estimated to be $1.79 million. This constitutes about 25.2% of the initial gross present value (GPV) of the project. However, the value decreased to about 21.3% of the initial GPV when the initial cost of investment is increased at the risk free rate over time (model 2). The option to defer value is highest in the discrete dividend adjustment with constant initial cost of investment (model 3), constituting about 28.3% of the initial GPV of the project. Again the value of the option decreased to $1.72 million (model 4), about 24.2% of the GPV, when the initial cost of the investment is increased at the risk free rate over time. The decrease in the option value as result of the increase in the initial cost of the investment is consistent with the theory of option pricing. This is because the option to defer is valued as an American call option and this type of option becomes less valuable as the exercise price goes up, all things being equal.

For this analysis, the ENPV[32] of the project with the flexibility to defer alone significantly improves from its negative value under the NPV without flexibility model to a positive value for all the four models and as result making the project economically attractive. This implies that there is value to waiting for the commencement of the hog investment project within the next 5 years. This is because the waiting period could lead to improved market conditions as well as a better strategy and decision making by management. This also implies that rejecting the project now, as indicated by the NPV approach, would not be an optimal decision. This result is consistent with that of Maung and Foster (2002). Maung and Foster (2002) used the real option approach to determine the impact of alternative marketing contracts on a decision to invest in a co-operatively owned hog facility. They concluded that between alternative marketing methods the option to defer constituted between 20-36% of the initial investment cost.

The results of this case study also indicate that the value of the option to defer in this analysis is affected by the type of dividend adjustments adopted. The option values were high under the discrete dividend adjustment compared to the continuous dividend adjustment approach, all things being equal. The possible explanation to this pattern of results is because in models 1 & 2 the project was assumed to pay dividends right from the beginning of the project even when the expected free cash flows were negative. Models 3 & 4 however account for the effects of dividend only when the expected free cash flows are positive. Thus, models 1 & 2 may be paying more dividends during the life of the options compared to models 3 & 4. Since the defer option is valued analogous to a call option and dividends reduce the value of call options, the pattern of the results are consistent with option pricing theory. For this project therefore, model 1 & 2 may be over accounting for the effects of dividends and as result may be underestimating the option to defer value given that free cash flows are not expected to be positive until the second year.

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[32] The ENPV for this section and subsequent sections represents the NPV plus the value of the option under which the discussion is being made. For instant in this section, ENPV = NPV + Defer Option

5.4.6 Option to Expand

The option to expand gives management the chance to benefit from favourable future market conditions. The option to expand the hog investment by 50% is also valued analogous to an American call option with an exercise price equal to the additional investment cost incurred for the expansion. The cost of the expansion is incurred from the purchase of sows and boars, cost of building and equipment, and the additional operating cost. The additional investment required for the expansion is estimated to be $4.18 million. The calculation of the additional cost of investment is presented in Table 5.6. The cost of expansion does not include the purchase of any additional land because it is assumed that the project already has enough land for the expansion. This option gives management the right but not the obligation to expand the pork operation if market condition is favourable, by investing $4.18 million any time during the next five years. Similar to the option to defer, this option is valued by first calculating the intrinsic values of the option at the end nodes of the binomial tree using equation 4.2. Management would only expand the production capacity if the project value ends up in any of the end nodes that has an intrinsic value of greater than zero, else management will not expand. The intrinsic values of the option to expand the hog project in the uppermost, second uppermost and down end nodes are respectively calculated as follows:

$$\text{Max } [0.5*\$919.24\text{M} - \$4.18\text{M}, 0] = \$455.44\text{M}$$

$$\text{Max } [0.5*\$622.95\text{M} - \$4.18\text{M}, 0] = \$307.30\text{M}$$

$$\text{Max } [0.5*\$0.05\text{M} - \$4.18\text{M}, 0] = 0$$

Thus, if the project ends up in the uppermost node, management will benefit from expanding the production capacity of the project. On the other hand, it would not be optimal for management to expand if the value of the project ends up in the down node. Moving a step backwards to period $t = 24$, equation 4.3 is used to determine if it will be optimal for the option to be exercised or kept alive. Using the uppermost value at this time period, the value of the option if exercised and the value of the option if kept alive are respectively calculated as follows:

$$0.5*\$756.73\text{M} - \$4.18\text{M} = \$374.18\text{M}$$

$$= e^{(-0.04*0.2)}[0.45*\$455.59M + 0.55*\$307.30M] = \$370.98M$$

$$\text{Max } [\$374.18\text{M}, \$370.98\text{M}] = \$374.18\text{M}$$

From the values calculated, the optimal decision for management is to expand the project if its value ends up in the uppermost node at $t = 24$. This is because the exercise value of the option at this node is greater than its value if kept alive. Repeating the backward calculation through the remaining nodes of the tree and making similar comparison and decision at each node results to the value of the option to expand the hog project at time period $t = 0$.

With the flexibility of expanding the pork project any time within the next five years, model 1 estimated this option value to be $0.92 million, which is approximately 13% of the initial GPV. Similar to the option to defer, the value of the option to expand decreased to about 11.2% of the GPV when the cost of expansion is increased at the risk

free rate over time. Using model 3, the value of this option to the hog project is estimated to be $1.04 million, about 14.7% of the initial GPV. The value decreases again under model 4 to just 12.5% of the initial GPV. Although the ENPV of the project improves when the option to expand alone is considered for the hog project compared to the static NPV, the ENPV is still negative irrespective of which model is used, making the project economically unattractive. For this project therefore, the option to expand the production capacity if market conditions improves although valuable, it still does not make the project attractive enough for management to undertake. The pattern of the results from the four models is similar to that of the option to defer. The reasons for this pattern of results are similar to those discussed under the option to defer since the option to expand is also valued analogous to an American call option. However, it seems the effect of the dividend adjustments is not as significant as it was with the option to defer. This difference is because the exercise cost of the option to expand is lower compared to that of the option to defer and as a result the increasing effect on their respective values would be minimal for the expand option than the option to defer. Also since the impact of both increases in the value of the exercise cost and dividends paid during the life of the option have negative effects on the value of call options, the pattern of results for the option to expand is as would be expected from option pricing theory.

5.4.7 Option to Abandon

The option to permanently abandon the hog project in exchange for its salvage value is valued analogous to an American put option with an exercise price equal to the salvage value. This option gives management the right but not the obligation to abandon the pork project any time within the next five years with an exercise price equal to the salvage value of the project. The salvage value is realised from the sale of the building and equipment, which is assumed to be worth 25% of its original value in the next five years, the sale of the land at its current value less 10% transaction cost and the sale of the hogs and boars as culled animals. It is also assumed that management will recoup the initial operation cost in full if the project is abandoned. The salvage value for the hog project is estimated to be $3.83M. The calculation of the Abandonment value is presented in Table 5.7. Using equation 4.4, the intrinsic values for this option at the uppermost and the down end nodes of the binomial tree are calculated respectively as follows:

$$\text{Max } [\$3.83\text{M} - \$919.24\text{M}, 0] = \$0$$

$$\text{Max } [\$3.83\text{M} - \$0.05\text{M}, 0] = \$3.78\text{M}$$

With this option, if the intrinsic value is greater than zero, management will abandon the project for its salvage value. On the other hand, management will hold on to the project if the intrinsic value of the option is zero. From the above calculations, the optimal decision for management is to abandon the hog project if its value ends up in the down node. However, if the project value ends up in the uppermost node, the project should not be abandoned. Similar calculations are used to obtain all the intrinsic values at the end nodes of the binomial tree. Using the backward technique to determine whether the option should be exercised in the nodes at time period $t = 24$, equation 4.5 is used. Using the down node at this time period, the value of the option if exercised and its value if kept alive are calculated below:

$$\$3.83\text{M} - \$0.07\text{M} = \$3.76\text{M}$$

$$= e^{(-0.04*0.2)}[0.45*\$3.74M + 0.55*\$3.78M] = \$3.73M$$

Max [\$3.76M, \$3.73M] = \$3.76M

Thus, the optimal decision for management would be to abandon the project if its value ends up in this node since the option value if exercised is greater than its value when kept alive. The backward calculation is repeated through the remainder of the nodes in the tree. The value of the option to abandon the hog project is obtained at the first node at time period $t = 0$.

The results of the option to abandon the hog project are presented in Table 5.5. The option to abandon the hog project any time within the next five years using the continuous dividend and constant abandonment value approach (model 1) is valued to be \$0.67 million, which is about 9.5% of the initial GPV of the project. The value of the option to abandon increased to about 14.5% of the initial GPV when the abandonment value is increased at the risk free rate over time (model 2). The value of the option to abandon when evaluated with discrete dividend and constant exercise value (model 3) is very close to the value obtained in model 1, approximately \$0.70 million. Similarly, the value of the option given by model 4 is also close to the value obtained from model 2, about 14.5% of the initial GPV of the project. The results indicate that the option to abandon the hog project is not affected much by the kind of dividend adjustments adopted. However, the nature of the abandonment value (constant or variable) had a significant impact on the value of this option. The value of the option to abandon the hog project significantly increased when the exercise value is increased at the risk free rate.

This result is consistent with option pricing theory because as the abandonment value goes up the opportunity to abandon the project becomes more attractive to management. The reason for this is because the option to abandon is valued as a put option and the value of put options respond positively to an increase in the exercise price. However, including the option to abandon alone in the project's ENPV does not make the project economically attractive. Although the value improved significantly compared to the NPV without flexibility case, the value was still negative for all four models. The reason for the low values of the option to abandon the hog project is due to the low salvage value realized from the abandonment. With the salvage value being low compared to the gross present value implies that the option would be out of the money in most parts of the binomial tree. This also implies that most of the nodes in the binomial tree will have zero intrinsic value and this will subsequently affect the overall value of the option to abandon the hog project. Because of the low salvage value, this option should not be particularly attractive to the management of the hog project if market and/or production conditions turn out to be unfavourable. However, if the unfavourable market conditions persist for a longer period of time and there are no signs of any improvement in the near future this option could be valuable. It would save management from incurring substantial losses due to the persistent poor conditions.

5.4.8 Multiple Option

As explained earlier, the value of an option in the presence of others may differ from its value in isolation and as a result necessitate the need to value the options in combination. Thus, the values of the options to abandon and expand were evaluated in

combination for the hog project. By following Copeland and Antikorav (2001), the combined value of the option to abandon/expand is evaluated by treating the options as a mutually exclusive decision at each node, and the decision that results in the highest payoff is chosen as optimal. The terminal payoffs of the combined value of the options to abandon and expand at the uppermost and down end nodes are respectively calculated using equations 4.6 as follows:

$$\text{Max } [0.5*\$919.24M - \$4.18M, \$3.83M - \$919.24M, 0] = \$455.44M$$

$$\text{Max } [0.5*\$0.05M - \$4.18M, \$3.83M - \$0.05M, 0] = \$3.78M$$

From the two equations above, the first terms represent the value of the option to expand whilst the second term is the value of the abandon option. From the comparison, the optimal decision for management of the hog project would be to expand if the value of the project ends up in the uppermost node of the tree. This is because the value of the expand option is highest at this node. Similarly, the value of the option to abandon is highest at the down end node and therefore the option decision for management is to abandon the hog project if its value ends up in the down end node. Similar calculations are used to obtain all the intrinsic values at the end nodes of the binomial tree. Using the backward technique to determine whether the project should be abandoned, expand or the option should be kept alive in the nodes at time period $t = 24$, equation 4.7 is used. Using the uppermost end node at this time period, the values of the options to abandon and expand if exercised are respectively calculated below:

$$0.5*\$756.73M - \$4.18M = \$374.18M$$

$$\$3.83M - \$756.73M = 0$$

The combined value of the option if kept alive is calculated as follows:

$$= e^{(-0.04*0.2)}[0.45*\$455.59M + 0.55*\$307.30M] = \$370.98M$$

The optimal decision is thus determined by comparing the above three calculated value and choosing the largest as follows:

$$\text{Max } [\$374.18M, \$0, \$370.98] = \$374.18M$$

Thus, the optimal decision for management would be to expand the project if its value ends up in this node within the maturity period of the options. The backward calculation is repeated through the remainder of the nodes in the tree. The combined value of the option to abandon and expand is obtained at the first node at time period $t = 0$. The results of the combined value of the abandon/expand options are shown in Table 5.5.

The combined value of the option to abandon and expand is highest in model 4, constituting about 27% of the initial GPV of the project, followed by model 2, about 25.6% of the initial GPV. The combined value of the two options was lowest in model 1 where the value constituted about 22.5% of the GPV. The high values obtained from models 2 & 4 indicate that the increase in the value of the option to abandon due to the increase in the abandonment value over time dominates the decrease in the expand option due to the increase in the cost of expansion over time. The values also show that the option to abandon and the option to expand for the hog project are additive since their combined value is the same as the sum of their separate values for all the models evaluated.

This result is consistent with the findings of Trigeorgis (1993a). Option values tend to be additive under the following two conditions: (a) when the options are of opposite type, that is, a call option optimally exercised when circumstances become favourable and a put option optimally exercised when circumstances becomes unfavourable and (b) the time of possible exercise of the two options are close together. The options to expand and abandon for the hog project are evaluated as call and put respectively with the same time to maturity. This satisfies the above conditions for option additivity. With the combined value of the options to expand and abandon, the ENPV of the hog project is positive. This implies that with the combined flexibility of expanding and abandoning the hog investment project some time in the future depending on the market conditions, the project would be viable to undertake immediately. If there is no defer option available, management can go ahead with the investment now if the combined flexibility of abandoning or expanding the hog project some time in the future exists.

From the results of the real option analysis discussed above, three main investment decisions can be made based on three different scenarios. The first scenario is by determining the value of the project with the option to defer the beginning of the project alone. The second scenario is where the value of the project is considered with the combined flexibility of the options to abandon and expand. The third scenario is where the values of the project from scenarios 1 and 2 are compared.

In the case of scenario 1 where the project value is determined based on only the option to defer the beginning of the project, the results indicate that the optimal strategy for the management of the hog project is to wait. This is because the value of the option to defer was greater than the NPV without flexibility. In addition, the ENPV of the project becomes positive for all four models when the defer option alone is considered, indicating that rejecting the project now would not be an optimal strategy for management. In scenario 2, the ENPV of the project is positive with the combined flexibility to abandon or expand (with no defer option) for all the four models indicating that the project is economically feasible to undertake. Under this scenario, the optimal decision for management is to go ahead with the investment right now if the abandon and expand options are the only options available for the project. By comparing the ENPV of the project with the combined flexibility to expand or abandon and the ENPV with the option to defer in the third scenario, two decisions were reached. When the exercise prices increase at the risk free rate (models 2 & 4), the results indicate that the optimal decision for the management of the project is to invest now. However, the models with constant exercise prices (models 1 & 3) indicate that the optimal decision for management is to defer the beginning of the project. This is because the ENPV with combined flexibility to abandon/expand is greater than the ENPV with the option to defer in models 2 & 4 and vice versa in models 1 & 3.

The results of the real option models seem to compare better to the actual decision taken by the management of the hog project since none of the scenarios suggested the project should be rejected. The NPV without flexibility suggested that the project should be rejected but the real options model suggested otherwise. Particularly, since the investors went ahead with the project, scenario 2 and the models with the increasing exercise prices (models 2 & 4) in scenario 3 are consistent with the decision of the management of the hog project. Scenarios 1 and the models with constant exercise prices

(models 1 & 3) in scenario 3 suggest that the optimal strategy for management would have been to defer the beginning of the project. Since the project was undertaken, the results form scenario1 and models with the constant exercise prices in scenario 3 are not consistent with the actual decision by the investors of the hog project. This inconsistency could be that either the estimated value of the option to defer in this analysis is higher than that perceived by the investors or that the option value had diminished by the time the investment took place.

Another issue related to this may be the optimal time for the options. The option to defer the project may not be available for 5 years or the options to abandon and expand may be available for a longer time period than 5 years. All the above factors could significantly affect the decision of the management of the hog project to start the investment now.

5.4.9 Sequential Compound Option

Two models are used to evaluate the compound sequential option for the hog project. The first is a continuous dividend adjustment with constant exercise prices and the second is a discrete dividend adjustment with constant exercise prices. In evaluating this option, the hog project was split into two investment phases. The first phase is the option to purchase the land after feasibility studies such as water testing or regulatory siting approval. It is assumed that the opportunity to invest in the first phase would exist for one and half years with an exercise price equal to the cost of the land, which is estimated to be $1.3 million. The second phase involves the option to invest in the project by constructing the barn and purchasing the breeding animals. The time within which management should decide on whether or not to exercise this building option is set at 5 years from the time feasibility study is conducted. The exercise price for the second option, which includes the cost of barn construction, cost of breeding animals and operating cost is estimated to be $7.4 million. Table 5.8 presents the input variables used for the estimation of the sequential compound option for the hog project. With this option, the first phase investment gives management the right but not the obligation to invest in the second stage if market conditions are favourable. If management decides not to proceed with the project after the first stage, the project could be abandoned and the land sold for its salvage value. The salvage value of the land is estimated to be $1.17 million, 10% below the original purchase value.

Using equation 4.8 and 4.9, the payoff values for the option to invest in the first and second stages of the investment are determined by choosing the net benefit of the investment or zero, whichever is larger. The next step is the calculation of the value of the option if exercised and its value if kept alive for each of the nodes in the tree using the backward technique discussed earlier.

With the continuous dividend and constant exercise prices, the value of the compound sequential option without the flexibility of selling the land for its salvage value after the first investment phase is estimated to be $1.23 million. However, when the managerial flexibility of abandoning the project after the first stage and sell the land for its salvage value was considered, the value of the option increased by about 32% to $1.62M. On the other hand, the discrete dividend adjustment with constant exercise prices without the flexibility of selling the land after the first stage investment estimated this option to be $1.43M. The value increased to $1.79M when the chance to abandon the

project and recover the salvage value of the land was considered, an increase of about 25%.

The value of the sequential compound option is interpreted as the present value of the project at pre-feasibility stage given that the hog project has two investment phases. Therefore if the cost of feasibility studies is greater than the present value of the project at pre-feasibility, then the project should be rejected. Thus, not only does this option help management to determine at pre-feasibility if it would be worthwhile to start the project given the cost of the feasibility studies, but also helps management to determine the maximum amount they should pay for feasibility studies such as licensing and other pre-investment investigations. Similar to the expand and defer options, the values of the sequential compound options were relatively higher when the discrete dividend approach was employed compared to the continuous dividend case. Again, this is due to the fact that the continuous dividend approach accounts for the effects of dividend right from the beginning of the project even when negative case flows are being incurred. Since the sequential option is valued as a compound call option, the impact of the dividend is to reduce the value of the option. Thus the pattern of results experienced in this section is consistent with option pricing theory.

5.5 Chapter Summary

This chapter presented and discussed the results of the quantitative valuation of the 2600 farrow-to-finish hog project in Western Canada. The hog project was first evaluated with the NPV without flexibility model and the result was negative. Thus with the static NPV model, the project would not be profitable to undertake and therefore it should be rejected. The results of the NPV model contradict the actual decision taken by the management of the hog project. The project was then evaluated by considering the impact of strategic managerial decisions on the project's value using the real options framework. In all, four different real options were identified as being valuable for the hog investment project. The four real options are defer, expand, abandon and compound sequential option. The valuation of the compound sequential option in this thesis was looked at in a different context than the remaining three other options. The sequential option determined the maximum amount management should pay for feasibility studies for the purchase of the land for the project.

The value of the option to defer the beginning of the project was quite valuable for the hog project. When this option alone is considered, the project becomes economically viable indicating that the optimal decision for management under this scenario is to defer the beginning of the hog project. Evaluating the hog project with the separate values of the options to expand or abandon did not make the investment opportunity economically viable to undertake immediately since the value of the investment was still negative in both cases. However, when the combined flexibility to abandon or expand the capacity of the project was considered, the ENPV under this scenario indicated that the optimal decision for management would be to invest in the hog project now. When the ENPV with the combined expand and abandon options was compared to the ENPV with the option to defer in scenario 3, two optimal decisions arose. For the variable exercise prices models, the optimal investment strategy for management is to invest now. This results and that of scenario 2 mirror the actual decision taken by the investors of the hog project. The constant exercise price models on

the other hand indicated that the optimal strategy for management is to defer the investment project. Given that the investors went ahead with the investment, this results and that of scenario 1 contradicts the investment decision of management. The reasons for this contradiction could be that the value of the option to defer might have diminished by the time the investment took place. Also, it could be that this analysis overestimated the value of the option to defer or that the time to maturity of this option was longer compared to what was perceived by the management of the hog project.

In terms of the different models used for option analysis, the pattern of the results compares quite well to what have been reported in the options literature. Consistent with the option pricing theory, the options to defer, expand and sequential compound options which are valued analogous to call options responded negatively to both an increase in dividend payments and the exercise prices. This implies that dividend payments and the kind of exercise prices used (constant or variable) in the estimation of the option values could play a role in management decision-making. If the dividend payments are treated as a loss of value due to the actions of competitors, the results buttress the point that perfect competition has a negative influence on the ability of investors to strategically delay their investment. This is implied by the decrease in the value of the option to defer when dividend is incorporated. On the other hand, the increasing exercise price and the dividend payment improved the attractiveness of the option to abandon the hog project after investing. This is because with the decrease in the project value due to dividend payment and the exercise price increasing at the same time, the odds of the project being abandoned increases. However, the impact of the dividend was not as significant as the increasing exercise prices.

In a different context, a compound sequential option was evaluated to determine the maximum amount management should pay for the cost of feasibility studies before the land purchase. The feasibility studies are necessary for various regulatory approvals before the project can go ahead. The results turned out that if the cost of feasibility studies is greater than 25% of the gross project value, the project should not be accepted. If the possibility of recovering the salvage value of the land after the first investment does not exist, the project should not be accepted if the cost of feasibility study is greater than 20% of the expected gross project value.

The real option analysis may give a more reasonable result compared to the actual decision taken by the management of the hog project. The static NPV model indicated that the project should be rejected but the real option analysis suggested otherwise. In the chapter that follows, a sensitivity analysis of the results is conducted to determine the impact of the various variables on the analysis results. Most of the sensitivity analysis is done using the results from the discrete dividend adjustment approach with constant exercise price (model 3). This is because model 3 fits more into the approach proposed by management to pay out a portion of the free cash flows to investors.

5.6 Tables for Chapter 5

Table 5.1 Calculation of the Initial Gross Present Value (V_0) for the 2600 Farrow-to-Finish Pork Project

Years	0	1	2	3 – 15*	Perpetuity**
Expected Revenue	$0	$2.33M	$8.61M	$116.70M	$59.84M
Expected Expenses	$0.57M	$4.27M	$7.10M	$95.80M	$49.38M
Free Cash Flows	-$0.57M	-$1.94M	$1.51M	$20.90M	$10.46M
Discounted F.C.F.	-$0.57M	-$1.69M	$1.14M	$6.90M	$1.29M
Total Gross Present Value *(V_0)*	$7.1M				

*The values in this period are obtained by summing the revenues and expenditures from the 3rd to 15th year in Excel.

**This value captures the perpetuity value of the project after 15 years

Table 5.2: Estimated Annual Discrete Dividend Yields for the Hog Project

Year	1	2	3	4	5
Dividend Yield	0%	6.14%	6.69%	6.72%	6.75%

Table 5.3: Hog and Barley Model Results of SUR Analysis from Historical Monthly Data from August 1991 to September 2002

Variables	Coefficients	Standard Errors	t-Ratios
$\beta_{0,Hog}$ ($/kg dressed)	0.38*	0.0760	4.96
$\beta_{0,Barley}$ ($/bushel)	18.73*	3.5770	5.24
$\beta_{1,Hog}$	0.77*	0.0467	16.51
$\beta_{1,Barley}$	0.87*	0.0265	32.68
Std Error of $\varepsilon_{,Hog}$	0.20		
Std Error of $\varepsilon_{,Barley}$	9.37		
Correlation Coefficient	0.21*		
System R-Square	0.95		
Likelihood Ratio	24.14		

*Statistically significant at 1%
Critical t-ratio = 1.96
Critical $\chi^2 = 6.64$

Table 5.4: Input Variables for the Valuation of the Options to Defer, Abandon and Expand the Hog Project

Input Variables	Initial	Value
Gross Present Value	V	$7.1M
Initial Investment Cost	I	$8.59M
Time to Maturity (Years)	T	5
Time Interval	Δt	0.2
Annual Volatility	σ	43.5%
Risk Free Interest Rate (Annual)	r	4%
Dividend Rate (Annual)	q	5%
Up Movement	u	1.21
Down Movement	d	0.83
Risk Neutral Probability – Div Adjusted	p	0.45
Risk Neutral Probability – Unadjusted	p	0.47
Additional Cost of Expansion	I'	$4.18M
Abandonment Value	I_A	$3.83M

Table 5.5: Types of Options and their Values under Each Model for the Hog Project

Option Type	Values			
	Model 1	Model 2	Model 3	Model 4
Option to Defer	$1.79M	$1.55M	$2.01M	$1.72M
Option to Expand	$0.92M	$0.80M	$1.04M	$0.89M
Option to Abandon	$0.67M	$1.02M	$0.70M	$1.03M
Multiple Option	$1.60M	$1.82M	$1.73M	$1.91M
ENPV (Defer Only)	$0.30M	$0.06M	$0.56M	$0.23M
ENPV (Expand Only)	$-0.57M	$-0.69M	$-0.45M	$-0.60M
ENPV (Abandon Only)	$-0.82M	$-0.47M	$-0.79M	$-0.46M
ENPV (with Multiple)	$0.11M	$0.33M	$0.24M	$0.42M

Model 1 = Continuous dividend adjustment with constant exercise prices.
Model 2 = Continuous dividend adjustment with exercise prices increasing at the risk free rate.
Model 3 = Discrete dividend adjustment with constant exercise prices.
Model 4 = Discrete dividend adjustment with exercise prices increasing at the risk free rate.

Table 5.6: Calculation of the Expansion Cost for the Hog Project

Activity	Amount
Cost of Sows and Boars*	$378,500
Construction Cost	$3,550,000
Operating Cost	$250,000
Total	$4,178,500

*1300 Sow to be Purchased @ $275 each and 14 Boars to be purchased @ $1500 each

Table 5.7: Calculation of the Abandonment Value for the Hog Project

Activity	Amount
Value of Land	$1,170,000
Barn Salvage Value	$1,775,000
Cull Sows and Boars	$383,000
Operating Capital	$500,000
Total	$3,828,000

Table 5.8: Input Variables for Sequential Option Valuation of the Hog Project

Input Variables	Initial	Value
Cost of Land	I_L	$1.3M
Cost of Construction/Breeding Animals	I''	$7.4M
Land Abandonment Value	L_A	$1.17M
Time to Maturity – 1st Stage (Years)	T	1.5
Time to Maturity – 2nd Stage (Years)	T	5
Delta t	Δt	0.25
Up Movement	u	1.24
Down Movement	d	0.80
Risk Neutral Probability- Unadjusted	p	0.47
Risk Neutral Probability- Div Adjusted	p	0.44

5.7 Figures for Chapter 5

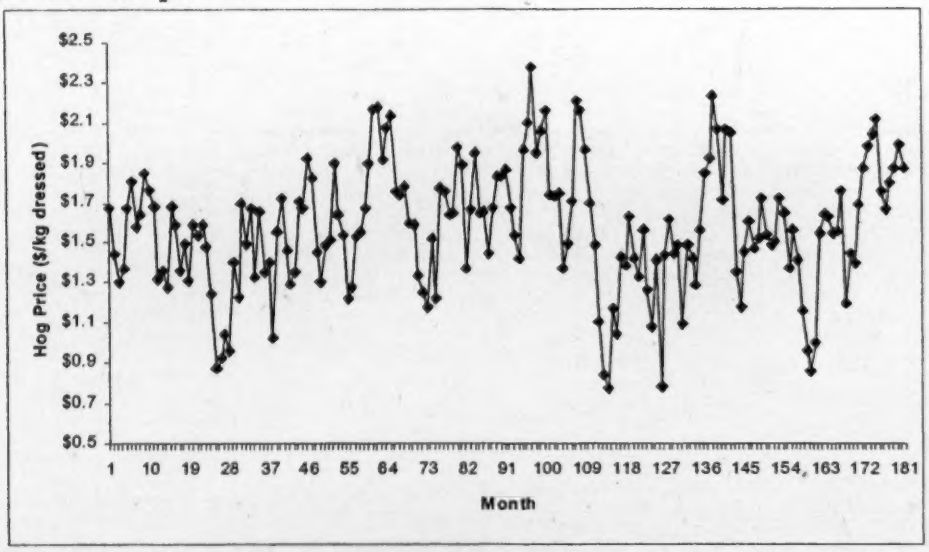

Figure 5.1: Predicted Mean Reverting Monthly Hog Prices for Monte Carlo Simulation Analysis

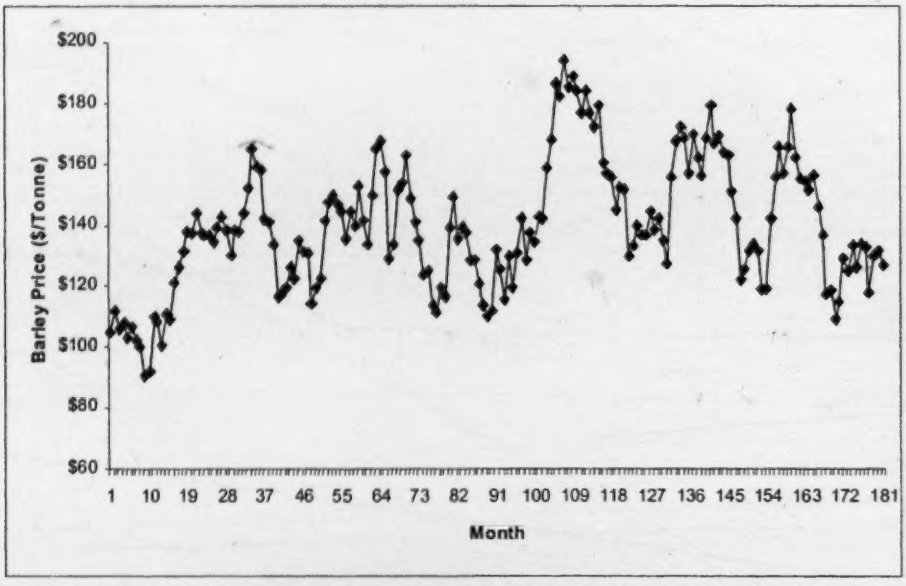

Figure 5.2: Predicted Mean Reverting Monthly Barley Prices for Monte Carlo Simulation Analysis

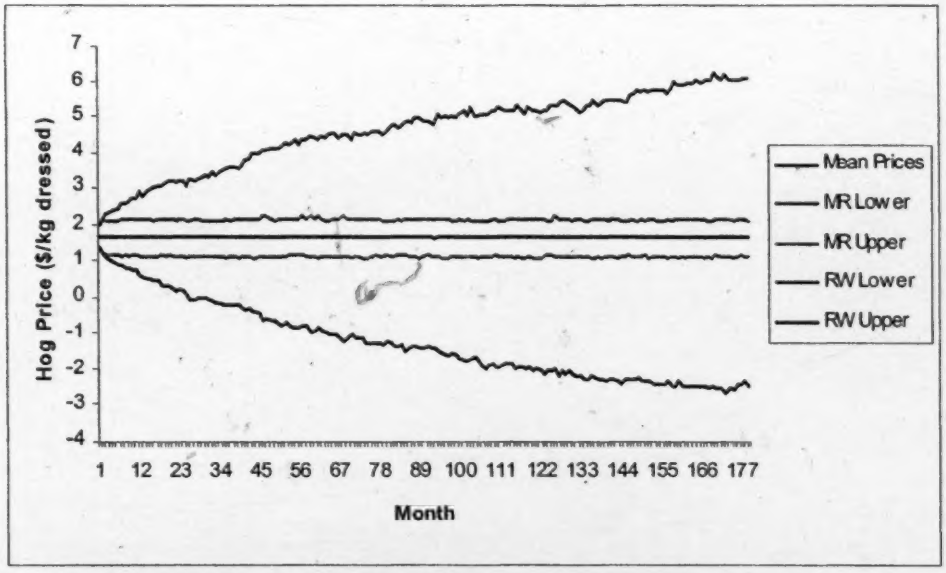

Figure 5.3: Comparison of Predicted Monthly Mean Reverting Hog Prices to Random Walk Hog Price at 90% Confidence Interval

MR = mean reversion and RW = Random Walk

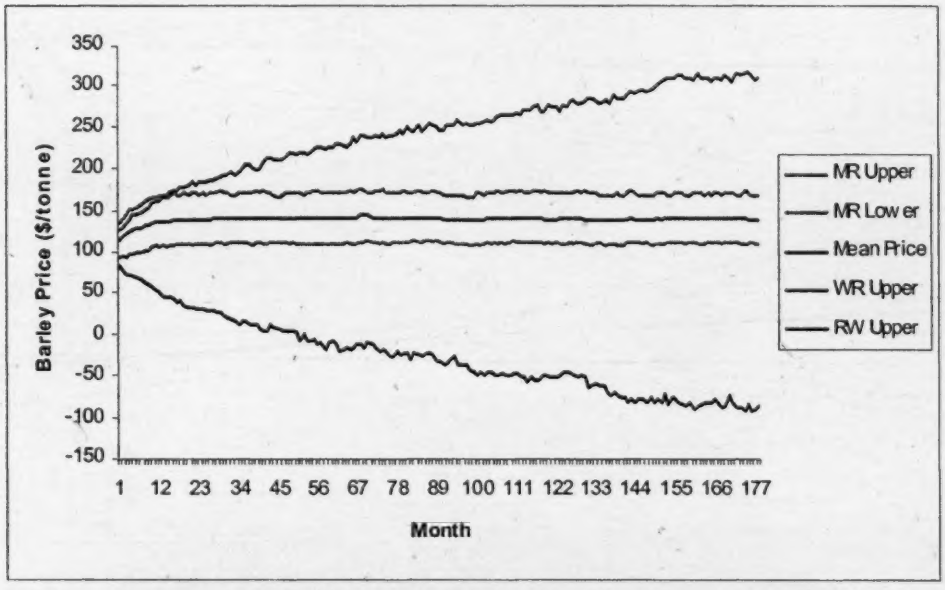

Figure 5.4: Comparison of Predicted Monthly Mean Reverting Barley Prices to Random Walk Barley Price at 90% Confidence Interval

MR = mean reversion and RW = Random Walk

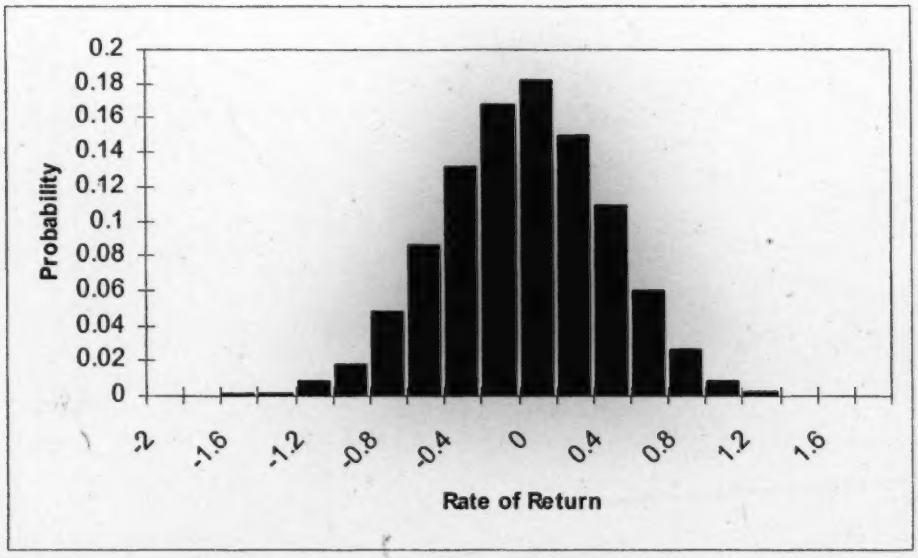

Figure 5.5: The Distribution of the Rate of Returns of the Gross Present Value of the Hog Project from Monte Carlo Simulation.

CHAPTER 6: SENSITIVITY ANALYSIS

Sensitivity analysis was performed to determine how sensitive the results of the 2600 farrow-to-finish hog project are to changes in the variables in the models. The sensitivity analysis examines the impact of a given variation in a key variable (one at a time) on the results of the analysis, holding all other variables constant. Several variables can be tested on their effects on the result of the analysis, but the sensitivity analysis was done on variables that are considered to be the most important in the analysis. In this chapter, the sensitivity analysis first determines how changes in variables such as output price and feed cost affect management decision to defer, expand or abandon. Second, the sensitivity of the NPV, the volatility of the project value and the gross present value of the project to changes in the discount rate are examined. Third, the analysis determines how sensitive the real option values are to changes in the gross present value of the project. Lastly, the analysis determines how sensitive the option values are under different gross project value to changes in the volatility.

The reasons why these variables were chosen for the sensitivity analysis is that: firstly, the discount rate is expected to have a significant impact on the NPV analysis, particularly for this project where the beginning years are not expected to return positive cash flows. Secondly, option values are to a large extent derived from the underlying asset's value and its volatility. As a result changes in these values are expected to have a significant impact on the results.

The sensitivity of the output and input prices to the decision of management (section 6.1) is conducted with all the four models from the real option analysis. The remainder of the sensitivity analysis presented in this chapter is done using only the results from the discrete dividend with constant exercise price approach (model 3) for the options to defer, expand and abandon. In the case of the sequential option, the discrete dividend adjustment approach is used. The reason why model 3 was chosen was discussed in chapter 5.

6.1 Sensitivity of Output and Input Prices

In this section, the analysis determines the impact of changes in the expected average output and input prices on management decision to defer, invest now or reject the hog project. In carrying out this sensitivity analysis, the static NPV model outputs of the gross project value *(V)* and the initial cost of the investment *(I)* were tied to the real option model when varying the expected average input or output price. As a result the option values change at the same time as the price of input or output is being changed. All the four models were used for the sensitivity analysis in this section and the results are presented in Table 6.1.

First, the analysis determines how sensitive the value of the option to defer is to changes in expected average hog or feed prices. This was done by changing the expected average price of hog or feed to equate the value of the defer option to the static NPV. The results determine the expected average prices of hog or feed at which it does not pay for management to defer the beginning of the project, all other variables in the model remaining the same. From model 1, the results indicate that if the expected average price of hog increases by 29% from the current price of $1.67 to $2.16 per kg dressed, it would make economic sense for management to start the project now, all things being equal.

The value was slightly lower in the case of model 2 where the result indicates that at an average expected hog price of $2.04 per kg dressed, an increase of about 23%, management should invest now. If the price of feed decreases from its current value of $74 to approximately $32 per finished animal or less in the case of model 1, a 56% decrease, the project should begin now. In the case of model 2, the result shows that at a feed cost of $41 per finished animal or less management can start the project now, a slightly higher value than that of model 1.

The above sensitivity analysis implies that the expected average prices of input and output play a role in management decision to either defer the beginning of the project or invest now. The expected average output and input prices at which the project should start now were respectively higher and lower in model 1 than in model 2. The pattern of the results is expected because the difference between the option to defer and the static NPV is higher in model 1 than it is in model 2. Model 2 has the exercise price increasing at the risk free rate and therefore reported a relatively lower option value than model 1 which uses a constant exercise price. The expected average price of hog at which the project should start now from this analysis is quite high compared to the historical price pattern of hogs in Alberta. A historical data of hog prices from 1991 to 2002 obtained from Alberta Agriculture shows the average price of hog for the period to be lower than the expected average price at which the project should start now from this analysis. This indicates that the estimated expected average price of hog (over $2/ kg dressed) at which the project should start now is not likely to be realized given the history of Alberta hog prices. Although hog prices have periodically topped the $2/ kg dressed mark in the past, this has only been in the short term.

The analysis tried to determine the expected average output price and feed cost at which it does not pay for management to wait using models 3 and 4. However, the input and output prices at which the project should start now could not be determined in the discrete dividend models (models 3 and 4) because of the relatively fewer dividends in the model. The fewer dividends in the discrete models result from the fact that the project is not paying dividends in the early years of the options life due to the negative cash flows returned by the project at that time[33]. This is related to the fact that it is usually optimal to exercise American call options just before the dividend paying period. However, the discrete dividend models are not accounting for dividends during the beginning time periods of the project. Therefore the input and output prices at which the project should begin now could not be determined by model 3 and 4. This results possibly suggest that the use of the real options approach to determine the timing of a non-dividend paying investments may not be appropriate.

Given that management has the combined flexibility to expand or abandon the project at any time within the next five years after investing, the analysis further determines how changes in hog or feed price would affect the decision to defer the hog project. In calculating these prices, the NPV without flexibility plus the combined value of the option to abandon and expand is equated to the value of the option to defer by changing the expected average market price of hogs or feed. The result of the analysis is presented in Table 6.1. The expected average price of hogs at which the project should start now was highest in model 3. The results indicates that, with the combined flexibility

[33] Note that when the models were tested with hypothetical dividend yields for the entire life of the option, the sensitivity analysis was able to determine the optimal values for feed and hog prices.

to abandon or expand, if the current expected price of hog increases by just 6% from $1.67 to $1.77 per kg dressed or more, it will be economically sensible for management to start the project now, all things being equal. Model 1 closely follows model 3 with an expected average hog price of $1.76 per kg dressed with model 2 recording the lowest expected average price of $1.73 per kg dressed. Similarly, the expected average cost of feed at which the project should start now given the expand/abandon options was lowest in model 3, closely followed by model 1 with model 2 recording the highest feed price. In the case of model 3, if the expected cost of feed decreases by 12% from $74 to about $66 or more a decision by management to invest now would make economic sense, all things being equal. The expected average cost of feed at which management should invest now for models 1 and 2 were $66.35 and $68.32 per finished animal respectively. This represents a decrease of about 11.5% and 8.3% for models 1 and 2 respectively compared to the initial feed cost of $74 per finished animal.

The expected average prices of both hog and feed were close in models 1 and 3 while that of models 2 and 4 were also close. This implies that the dividend adjustments did not have much of an impact on this sensitivity analysis but the increasing exercise prices did. Again, the results reflect on the difference between the ENPV of the project with the expand/abandon options and the option to defer. The bigger the difference in value the bigger the change in price, all things being equal. As expected however, the change in hog and feed prices were smaller when the project value included the combined flexibility of abandon and expand compared to when only the static NPV was considered. This is because when the combined flexibility is considered the ENPV of the project improved significantly and as the project value improves the attractiveness of the defer option decreases. The expected average prices of hog at which the project should start now from the analysis given the combined flexibility of the abandon/expand options are still not consistent with the historical price pattern of hogs in Alberta. The average historical price of hogs in Alberta is lower than the prices determined by the analysis indicating that these prices may not be practically attainable in the long term.

It is important to realize that similar analysis could be conducted to determine the input and output prices at which the project should start now using the separate values of either the option to expand or abandon. However, the ENPV of the project with either the option to abandon or expand alone is negative and therefore this analysis was not carried out in this thesis. Also the ENPV with the combined flexibility to expand/abandon can be compared to the ENPV with the option to defer to determine the expected average output and feed price at which management should start the project now.

The sensitivity analysis further estimates the abandonment or salvage value at which the project should not be deferred. Using model 3, if management can abandon the project at any time and receive about $8.8M as salvage value of the project, then there is no need for management to defer the beginning of the project. This was closely followed by the estimated salvage value of $8.5M from model 1 with model 2 recording the lowest salvage value at which the project should not be deferred. The above results imply that if the decision to invest in the pork project is almost completely reversible, the option to defer has no value. This is because the impact of downside risk on the value of the project is almost non-existent since management would simply abandon the project and recover the investment cost if market conditions deteriorate. This result is consistent with the real

options literature which indicate that the option to defer is valuable when a project is irreversible.

In a similar fashion, the expected average output and feed prices at which management should reject the project now were determined and the results are presented in Table 6.1. The results from model 3, which is the lowest, turn out that if the expected average price of hog decreases by 4% from $1.67 to about $1.61 per kg dressed or more the project should be rejected, holding all other variables constant. Model 2 had the highest price and it indicates that if the expected average price of hog decreases by less than 2% from $1.67 to $1.65 per kg dressed, the hog project should be rejected. On the other hand, model 3 indicates that if expected average feed cost increases by about 7% from $74 to about $79 per finished hog sold or more, it would make economic sense for management to reject the pork project, all things being equal. The expected average feed price at which the project should be rejected was lowest in model 2 where an increase of just 1.4% from $74 to $75 per finished hog should be enough for the project to be rejected. The change in hog and feed prices at which the project should be rejected is not consistent with the information from the case study and the historical hog price pattern in Alberta.

Information from the case study indicates that with an average input costs, existing farrow to finish hog projects are manageable with average finished hog prices as low as $1.50 per kg dressed. This is also backed by the historical price pattern of hog in Alberta. The reason why the expected average prices at which the project should be rejected are low is because the NPV without flexibility is already negative and as a result a further decrease in output price or an increase in feed cost is expected to have a significant negative impact on the decision of management, all things being equal. One reason for the negative NPV for the hog project may be due to the discount rate used for the analysis. The discount rate chosen for the analysis was based on the general risk level of hog production and as a result may not exactly reflect the risk level of this project. When the discount rate is decreased to 12%, expected average hog and feed prices at which the project should be rejected were much lower and higher respectively, compared to when the discount rate was 15%. The results indicate that if average expected hog price decreases to between $1.49 and $1.52 per kg dressed from the initial price of $1.67 per kg dressed, the project should be rejected. On the other hand if the expected average cost of feed increases by between 18% to 21% from $74 to between $87 and $90 per finished animal, the project should be rejected. To have an idea of the impact of the discount rate on the NPV without flexibility and the other variables in the model, a discount rate sensitivity analysis is conducted in the section that follows. The sensitivity analysis in the next section and the subsequent sections are performed using the results from model 3.

6.2 Risk Adjusted Discount Rate Sensitivity

The risk-adjusted discount rate is a measure of the degree of risk associated with the project. Higher discount rates are associated with high-risk investments and vice versa. Thus the discount rate plays a key role in determining whether an investment opportunity is worth to undertake or not. It is therefore important to determine how sensitive the results are to different discount rates. As a result, a sensitivity analysis is

conducted to determine the effects of the risk-adjusted discount rate on the project's NPV, gross present value and volatility of the project's value.

To determine the impact of the discount rate on the NPV results, the discount rate was varied from 0% to 40% and the corresponding values of the static NPV observed, holding all other variables constant. Figure 6.1 presents the results of the sensitivity analysis of the discount rate with respect to the NPV and the volatility of the project value. The results indicate that the NPV is quite sensitive to the discount rate. As can be seen from Figure 6.1, there is an inverse relationship between NPV and the discount rate. An increase in the discount rate leads to a decrease in the NPV, holding all the remaining variables in the model constant. This is consistent with the traditional capital budgeting theory. The NPV is more sensitive to the changes in the discount rate initially and the impact decreases as the discount rate increases. For instance, when the discount rate decreases by about 50% from 15% to 8%, NPV increases from its negative value of $-1.49 million to about $6.94 million (over 500% increase). On the other hand, the NPV went down from $-1.49 million to $-4.59 (a decrease of about 300%) when the discount rate increased by 60% from 15% to 24%. The shape of the curve is consistent with the discounting process in NPV. The discounting in NPV is geometric and therefore the shape of the curve is not surprising. As expected, the results indicate that as the degree of risk associated with the project increases, management will need more compensation in the form of expected returns in order to undertake the project. The project's static NPV is positive when the discount rate is 12% or less. This implies that the results of the NPV will be consistent with the actual decision taking by the management of the hog project if the investor discount rate is 12% or less.

In determining the impact of the discount rate on the volatility of the project value, the discount rate in the stochastic NPV set up was changed in increments of 4% from 0% to 40%. Each time the discount rate is changed the initial gross project value (V_0) changes and the Monte Carlo simulation is run again to determine the corresponding volatility of the project value[34]. As can be seen from Figure 6.1, the discount rate had a positive effect on the volatility of the hog project returns. An increase in the discount rate leads to an increase in the project's value volatility. This indicates that as the risk level associated with project increases so too does the volatility surrounding the gross present value of the project, all things being equal. The impact of the discount rate on the volatility was however higher at the lower rates and diminishes as the rate increases. For example, an increase in the discount rate from 15% to 20% leads to an increase in the volatility of the project value from 43.5% to 86%, close to a 100% increase. However, when the discount rate increases from 20% to 24%, the volatility increases by just 40% (from 86% to 120%).

The positive relationship between the discount rate and the volatility of the project value found in this analysis is consistent with that of Davis (2002). Davis (2002) used the concepts behind the CAPM and CML to explain the relationship between discount rate and volatility in real options analysis[35]. Both the CAPM and CML indicate that if an asset exhibits a systematic risk, which is measured by the volatility of the project, the investor would require higher returns to compensate for the risk. That is, the higher the level of the systematic risk associated with the project, the higher the returns (measured by the

[34] See section 5.3 on how Monte Carlo simulation was used to determine the volatility of the project.
[35] See section 2.1.2 for the discussion on the CAPM and CML models.

discount rate) investors would demand from the project. Therefore as the volatility of a project increases the expectation is that the discount rate should also increase and vice versa, all other things being equal. This implies that the discount rate and the volatility are not independent but closely related. Therefore if the volatility is being changed then perhaps it is necessary to change the discount rate at the same time if meaningful results are to be obtained.

The sensitivity of the gross project value to changes in the discount rate is similar to that of the NPV. All things being equal, as the discount rate increases the gross project value decreases, showing an inverse relationship between the two variables. Figure 6.2 depicts the relationship between the discount rate and the gross present value of the project. The impact of the discount rate on the gross project value is high at lower discount rates but as the discount rate increases the gross project value decreases at a decreasing rate. For example, when the discount rate increases from 15% to 20%, the gross project value decreases by about 35% from $7.1 million to $4.63 million. However, the gross project value decreases by only 25% (from 4.63 million to $3.45 million) when the discount rate increases from 20% to 24%. The shape of the curve is related to the geometric type discounting of the NPV model. The result implies that the project's value goes down when the level of risk associated with the project increases. This is consistent with the theory of traditional capital budgeting methods. However, this is where the real option pricing theory diverges from the traditional methodology. With the option pricing theory, a higher project risk, driven by the volatility of the project value does not necessarily decrease the value of a project, all things being equal. This issue is discussed further in section 6.4.

6.3 Gross Project Value Sensitivity

The gross project value (GPV) is calculated from the future expected cash flows (excluding the initial cost of the investment) of the project. Because these cash flows are the expected earnings in the future, they are difficult to predict. Several factors such as action taken by competitors, uncertainty of the market place, among others contribute to the difficulty of predicting the GPV. This section analyses the effects of changes in the GPV on the separate values of the real options for the hog project. In carrying out the sensitivity analysis in this section, the volatility was held constant at 43.5%.

Figure 6.3 presents how changes in the GPV affect the value of the option to defer, option to expand and the option to abandon the hog project. The sensitivity of all the option values to changes in the GPV is consistent with what would be expected from standard option pricing theory. The results show a positive relationship between the option to defer and the GPV. That is, as the GPV increases the value of the option to defer also increases, with all other factors held constant. The reason is because the option to defer is valued analogous to a call option and call option value increases as the value of the underlying asset increases. The value of the option to defer the hog project is sensitive to the value of the project, as indicated by the steeper slope of the graph. When the GPV is doubled from $7.1 million to $14.2 million, the value of the option to defer more than doubles (from $2.01M to $7.3M), with all other variables in the model held constant.

The sensitivity of the value of the option to abandon the hog project to changes in the value of the GPV shows a negative relationship. This is also consistent with the

theory of option pricing since the option to abandon is valued analogous to a put option. The slope of the value of the option to abandon appears to be steep initially but gradually declines as the GPV increases further. The downward sloping nature of the value of the option to abandon the hog project implies that as the project value increases, the attractiveness of the option to abandon the project decreases. For instance, when the GPV is $30 million the value of the option to abandon was just $0.06 million compared to its base case value of $0.74 million when the GPV is $7.1 million. The reason is that with the initial investment cost held constant and the GPV increasing, the intrinsic values of this option fall further and further out of the money and as a result the value of the option decreases.

As can be seen from Figure 6.3, there is a positive relationship between the option to expand and the GPV. Just like the option to defer, the expand option is also valued analogous to a call option and as a result an increase in the value of the underlying asset is expected to have a positive impact on its value. The value of this option is also quite sensitive to the GPV. The base case value of the option to expand was $1.04 million when the GPV is $7.1 million. However, when the GPV is $30 million, the value of the option to expand increased to over $11 million. This implies that expanding the pork project becomes a very attractive option to management as the project's value goes up.

With the GPV having opposite effects on the separate values of the option to abandon and expand, the impact of the GPV on the combined value of both options is also evaluated. The sensitivity of the combined value of the option to abandon and expand to changes in the GPV is presented in Figure 6.4. The slope of the curve shows a positive relationship between the combined value of the option to abandon and expand and the GPV. This implies that the increase in the value of the option to expand due the increase in the GPV dominates the decrease in the value of the option to abandon, all things being equal.

The analysis further determines the sensitivity of the compound sequential options (with and without abandoning the project after the first stage) to changes in the GPV. The sensitivity analysis for this option is done using the results from the discrete dividend approach and the result is presented in Figure 6.5. An increase in the GPV makes both compound sequential options more valuable as is indicated by the upward sloping curve of Figure 6.5. This is as expected from option pricing theory because the sequential option is valued analogous to compound call option. However, the values of the sequential options appear to get closer and closer as the GPV increases. For instance, when the GPV is $7.1 million, the compound sequential option value when there is no land abandonment is $1.16 million and its value when there is land abandonment is $1.56 million. The values were virtually the same, $21 million and $21.1 million for abandonment and without abandon respectively, when the GPV is $30 million. This result from the fact that as the GPV increases, the option to abandon the project after the first stage investment becomes less and less valuable.

6.4 Volatility Sensitivity

The level of risk surrounding the value of the project in the real option analysis is measured by the volatility of the project value. It is therefore important to determine how sensitive the option values are to changes in the volatility of the gross project value. To analyse the effect of the option values to the volatility, a comparison of three different

GPVs are considered. The first is a base case scenario, which is the expected scenario for the project with a GPV of $7.1M (the actual analysis). The second is the best-case scenario where the expected GPV is assumed to increase by 30% from $7.1M to $9.23M. The third is a worse case scenario where it is assumed that the expected GPV is decreased by 30% from $7.1M to $4.97M. The purpose of this is to determine whether the volatility of the value of the project will have different effects on the projects value under the different scenarios.

Figure 6.6 shows how sensitive the option to defer is to changes in the volatility for the three scenarios. The curve shows that higher project value risk driven by the volatility increases the value of the option to defer the beginning of the hog project. The reason for the increase in the option value is because as the volatility increases, the spread of the future GPV becomes wider which in turn increases the chances of the option being deep in the money within the maturity date. Thus by deferring the beginning of the project, management can take advantage of upside movements while at the same time limiting the downside risk of the project. The results further show that all three scenarios are sensitive to the volatility of the project's value. However, it is interesting to note that the worse case scenario is more sensitive to changes in the volatility compared to the base case and best case scenarios. For instance, the option to defer value increases by less than five times ($1.37M to $6.3M) when volatility rises from 10% to 100% in the best-case scenario. In the case of the base case scenario, the option value increases by about 27 times (from $0.17M to $4.57M) when the volatility increases from 10% to 100%. The impact of volatility is greater in the worse case scenario where the option to defer value increase from virtually zero to $2.94M when the volatility rose from 10% to 100%. The sensitivity analysis results indicate that as the risk surrounding the value of the project increases, the impact on the option to defer the project is high when the GPV is low compared to when GPV is high, all other variables in the model remaining constant. This is as expected from the theory of option pricing. Ott and Thompson (1996), Trigeorgis (1993b, 1999), Schwartz and Moon (2000) and Copeland and Antikarov (2001) all found option values to be increasing with volatility.

The findings of Davis (2002) however challenge this generally accepted norm of option value increasing as the volatility of the project increases. Davis (2002) showed that an increase in volatility might even harm a call option value[36], particularly if the option is in the money. Again Davis (2002) based his argument on the relationship between the discount rate and volatility, indicating that there are two effects on call option values on a risky underlying asset when volatility increases. The first is a direct effect, which result from directly increasing the volatility in the real options model and observing its effects, as was the case in the analysis shown above. The second is an indirect effect which results from the fact that an increase in the volatility may also be associated with an increase in the discount rate. The implication of the second effects is to decrease the gross present value of the project and for that matter the value of the call option contingent on the gross project value. Therefore, an increase in volatility increases the value of the defer option through the direct effect but lowers the defer option value through the indirect effect. As a result, the total effect of an increase in volatility on call option values, such as the option to defer, is indeterminate. Directly changing the volatility in the real option model to determine its impact on the option values may fail to

[36] Davis (2002) uses a growth option in his example.

account for the indirect effects the volatility may have on the option values. This could lead to over estimation of the impact of volatility on the option values.

To evaluate Davis's (2002) argument, a sensitivity analysis was carried out to determine the corresponding discount rate and volatility for the best and worse case scenarios used for the analysis in this section. This was done by calculating the discount rates consistent with that given by the GPVs for the best and worse case scenarios, holding all other variables constant. This discount rate is then used to determine the corresponding volatility from Figure 6.1. With the base case scenario, the initial GPV of $7.1M corresponds to a risk-adjusted discount rate of 15% with a volatility of 43.5%. However, the discount rate would have to decrease from 15% to 12.4% for a GPV of $9.23M in the best-case scenario, all things being equal. At a discount rate of 12.4%, the volatility of the project value from the Monte Carlo decreases from 43.5% to 28%. On the other hand, for a GPV of $4.97 in the worse case scenario the discount rate would have to increase to about 20%. This discount rate corresponds to an increase in the volatility to 80% from the initial 43.5%. The different volatility for each of the scenarios with their corresponding option values are highlighted on Figure 6.6 with a circle.

The value of the option to defer increased by 18.4% in the best case scenario to $2.38M when the volatility and discount rate decrease to 28% and 12.4% respectively, compared to the base case scenario value of $2.01M. However, the value of the option was about the same as the base case when the volatility was increased to 80% with a corresponding increase in the discount rate to 20% in the worse case scenario. The implications of the above results are that an increase in the volatility may be associated with an increase in the discount rate thereby decreasing the GPV and vice versa. Thus, whereas an increase in the volatility of the project value would enhance the value of the option to defer in standard option analysis, the decrease in the GPV due to the increase in the discount rate would decrease the value of the option to defer. The total impact of an increase in the volatility on the value of the option to defer is therefore indeterminate. It would depend on the magnitude of the change from the direct and indirect effects. In the above example, the indirect effect dominated the direct effect in the best case scenario but both effects neutralized each other in the worse case scenario compared to the base case. This result is consistent with the arguments of Davis (2002).

An increase in the volatility of the project value results in an increase in the value of the option to abandon the hog project for all the scenarios, assuming all other variables are held constant. Figure 6.7 illustrates the sensitivity of the option to abandon value to changes in the volatility for the different scenarios. Like the option to defer, the reason for the increase in the value of the option to abandon is that an increase in the volatility of the project value increases the downside risk of the project and for that matter the chances of the project being abandoned. In the case of the option to abandon, the impact of the volatility is greater in the best-case scenario than the base case and the worse case scenarios. The option to abandon value increases from zero to $2.1M when the volatility rises from 10% to 100% in the best-case scenario. The impact of the volatility is lowest in the worse case scenario with the value increasing from close to $0.11M to $2.26M when the volatility rises from 10% to 100%. The results seem to imply that the effects of an increase in the volatility of the project value is greater for the option to abandon when the GPV is high than when it is low. The reason relates to the fact that with high GPV and low risk, the chances that the project would be abandoned is pretty much non-existent.

However, as volatility increases the downside risk of the project also increases and this increases the chances of the project being abandoned.

Figure 6.8 shows how sensitive the value of the expand option is to changes in the volatility under the different scenario. As can be seen from Figure 6.8, there is a positive relationship between the option to expand and the volatility of the project's value for all the scenarios, all things being equal. Like the option to defer, the impact of the volatility is greater in the worse case scenario and lower in the best-case scenario. The value of the option increases from virtually zero in the worse case scenario to $1.47M while the option value increases from $0.77M to $3.18M in the best-case scenario when volatility increases from 10% to 100%. This indicates that as volatility increases, the value of the option to expand the project increases faster when GPV is lower than when GPV is higher, all things being equal.

The result is consistent with the option pricing theory and the findings of Trigeorgis (1993b, 1999), Copeland and Antikarov (2001). But the findings of Davis (2002) challenges the result of the sensitivity analysis presented in this section for the reasons discussed earlier. Consistent with Davis (2002) however are the points highlighted on Figure 6.8 (with a circle) for each of the three scenarios. In this case, a change in the volatility corresponds to a change in the discount rate leading to a change in the GPV. Similar to the defer option, the indirect effect dominated the direct effect in the best-case scenario compared to the base case scenario. This is because the value of the option to expand increased from $1.04M in the base case to $1.28M in the best case when the volatility was decreased to 28% with a corresponding decrease in the discount rate to 12.4%. The value of the option to expand did not change in the worse case scenario when both the volatility and discount rate were increased compared to the base case.

Finally, the sensitivity of the compound sequential option to changes in the volatility of the project value is conducted on the discrete dividend approach with both the value of the option with and without abandonment after the first stage investment. This analysis uses only the base case scenario where GPV is $7.1M. The results are presented in Figure 6.9. Consistent with option pricing theory, the value of the compound options increases as the volatility surrounding the project value increases. The compound sequential option without abandonment increases from zero to $3.74M when the project value volatility increases from 10% to 100%. Similarly, the sequential option with abandonment increase from less than $0.01M to $4.24M when volatility rises from 10% to 100%. This is consistent with the results of Wilmot (2001) who states the sequential option to be increasing and sensitive to changes in volatility. The result is also consistent with option theory since the sequential option is valued analogous to a compound call option. What is interesting is that as the volatility increases, the difference between the values of the two compound sequential options (with and without abandonment) also increases. As can be seen from the graph, when the volatility is 10%, the option values from both scenarios are about the same. However, as the volatility increases the values of the option also grow apart with the option value with abandonment rising faster than the value with no abandonment. This is as a result of the fact that an increase in the volatility increases both the downside and the upside risks of the project. The result of this is to increase not only the chances of investing in the second stage investment but also the chances of abandoning the project after the first stage investment and recover the salvage value of the land.

6.5 Chapter Summary

This chapter focused on determining the sensitivity of the results from the hog project analysis to changes in the variables in the model. The sensitivity analysis examined a given variation in a key variable on the results holding all other variables constant. The results indicate that changes in the average expected prices of hog and feed could have a significant impact on management decision to either invest now or defer the beginning of the project. An increase in the average expected price of hog or a decrease in the average expected price of feed could tilt the decision of management in favour of investing now rather than deferring. This is particularly the case when the combined flexibility of the options to abandon and expand was included in the project value. The sensitivity analysis further shows that if the investment is almost completely reversible, the option to defer has no value to the management of the hog project. The results of the sensitivity analysis also indicate that a slight decrease or increase in the respective average expected prices of hog or feed would influence management to reject the hog project. However, the expected average prices determined by the analysis contradict the information provided by the case study and also the historical price patterns of hogs in Alberta. Thus, these prices may not be practically attainable for a long time given the past behaviour of hog prices.

The relationship between the NPV and the discount rate indicates that as the level of risk associated with the project increase, investors would demand more compensation in order to undertake the project. The results showed that, at a discount rate of 12% or lower, the results of the static NPV would be consistent with the decision taken by management of the hog project. The volatility of the project value estimated using the Monte Carlo and the discount rate showed a positive relationship. An increase in the discount rate leads to an increase in the volatility estimate of the project value. This relationship is consistent with the principles underlying the development of the CAPM and CML model. The implication of this result is that the discount rate and the volatility may not be independent. Therefore if the volatility is being changed then it may be necessary for the discount rate to be changed simultaneously in order to obtain meaningful option value results.

The impact of changes in the gross project value is also examined on the values of the real options. The defer and expand options showed a positive relationship to changes in the gross present value of the project. This is consistent with option pricing theory since both options are valued analogous to American call options. On the other hand, the option to abandon which is valued analogous to American put option reacted negatively to the increases in the gross project value. This is because with the initial cost of the investment held constant and the GPV increasing, the intrinsic value of the option to abandon falls further and further out of the money and therefore the attractiveness of this option decreases. Similar to the other call options, an increase in the GPV resulted in an increase in the value of the compound sequential option both with and without abandonment. However as the GPV increases, the value of the option to abandon the hog project after the first stage investment diminishes and the values of both sequential options (with and without abandonment) converge.

All the option values increase as the risk surrounding the value of the project goes up irrespective of the gross project value scenario. As the volatility goes up the values of

the options to defer and expand went up for all the three scenarios, holding all other variables constant. As expected, the results show the option values to be very sensitive to changes in the volatility of the project value. This is consistent with what have been studied in the real option literature. As volatility increases the spread of the future gross present value of the project becomes wider which in turn increases the chances of the options being deep in the money within the maturity date. However, directly changing the volatility in the option model to assess the impact on the option values may hide the indirect effects the volatility may have on the real option values. An increase in volatility may be associated with an increase the discount rate which in turn decreases the gross project value. The impact on the value of a call option may be indeterminate. However, most of the sensitivity analysis in this section is not accounting for the indirect impact of the volatility on the option values. Therefore the results may be over estimating the impact of the volatility on the values of the option to defer and expand. The option to abandon also had a positive relationship with changes in the volatility of the project value. This is because as the volatility increases the downside risk of the project also increases and so also is the chances of abandoning the project. The relationship between the values of the sequential compound option and the volatility was positive. However, since the sequential option is also valued as a compound call option the indirect effect of the volatility on its value is not captured by this analysis.

The sensitivity analyses reported in this chapter is consistent with what has been examined in both traditional capital budgeting and real options literature. The sensitivity analysis shows that the results of the NPV are consistent with the decision of the management of the hog project if the discount rate is 12% or lower. Similarly, an increase in hog price or a decrease in feed cost could favour investing now against deferring the project. However, the results of the sensitivity analysis should be interpreted with caution, particularly the impact of volatility on the values of the estimated options. The analysis has demonstrated that the volatility and the discount rate may not be independent. As a result changing the volatility while holding the discount rate constant may not account for the full impact of the change in the targeted option values.

6.6 Tables for Chapter 6

Table 6.1: Results of the Sensitivity of Option Values to Changes in Input and Output Prices

Description	Model 1	Model 2	Model 3	Model 4
Hog Price at which invest now	$2.16/kg dressed	$2.04/kg dressed	–	–
Feed cost at which invest now	$31.77 /finished animal	$41.36/finished animal	–	–
Hog price at which Invest now with Multiple Option	$1.76/kg dressed	$1.73/ kg dressed	1.77/ kg dressed	1.74/ kg dressed
Feed cost at which Invest now with Multiple Option	$66.35/finished animal	$68.32/ finished animal	66/finished animal	67.63/finished animal
Salvage value at which invest now	$8.50M	$6.90M	$8.80M	$7.02M
Hog Price at which abandon	$1.62 / kg dressed	$1.65/kg dressed	1.61/ kg dressed	1.64/ kg dressed
Feed cost at which abandon	$78/finished animal	$75/finished animal	79/finished animal	76.37/finished animal

Initial hog price = $1.67/kg dressed.
Initial feed cost = $74/finished animal
Multiple Option = the combined value of the options to expand and abandon.
Model 1 = Continuous dividend adjustment with constant exercise prices.
Model 2 = Continuous dividend adjustment with exercise prices increasing at the risk free rate.
Model 3 = Discrete dividend adjustment with constant exercise prices.
Model 3 = Discrete dividend adjustment with exercise prices increasing at the risk free rate.

6.7 Figures for Chapter 6

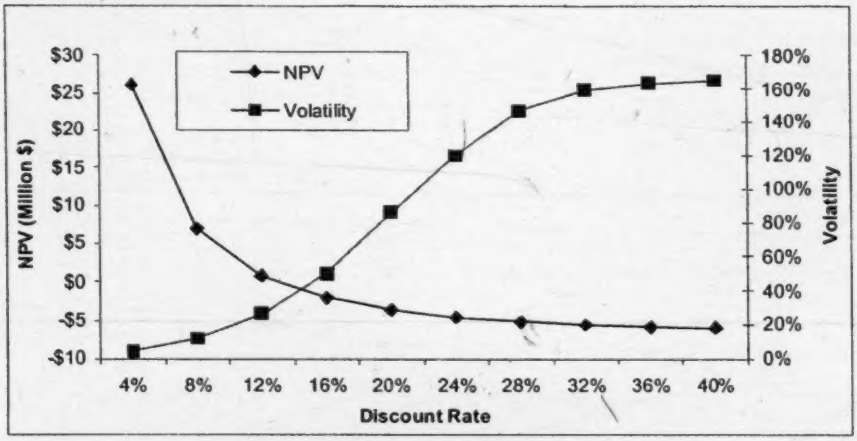

Figure 6.1: Sensitivity Analysis of the Impact of Discount Rate on NPV and Volatility of Project Value

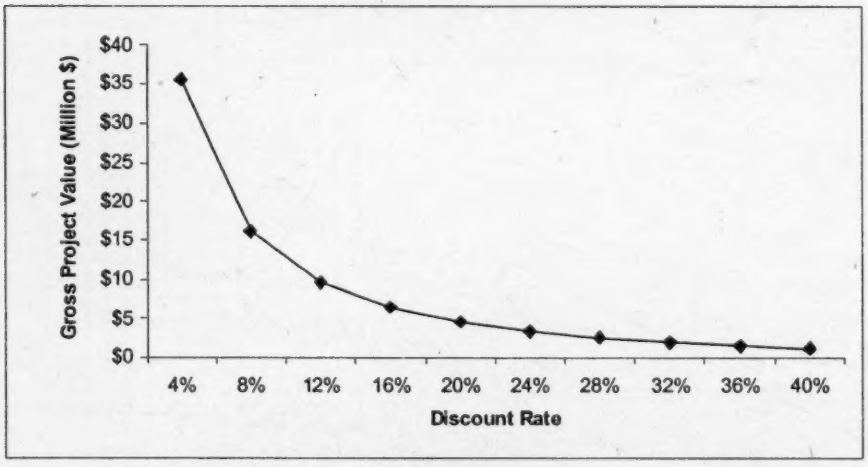

Figure 6.2: Sensitivity Analysis of the Impact of Discount Rate on Gross Project Value (V)

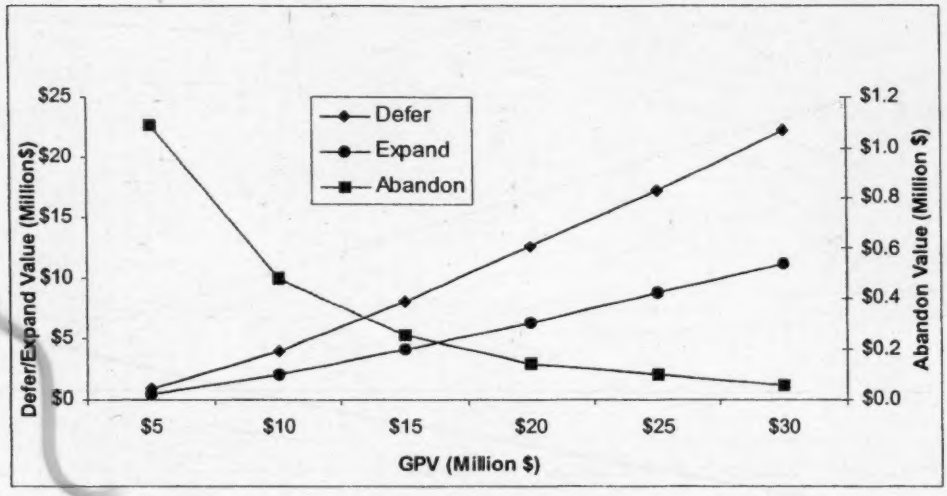

Figure 6.3: Sensitivity of Option Values to Changes in Gross Present Value (GPV)

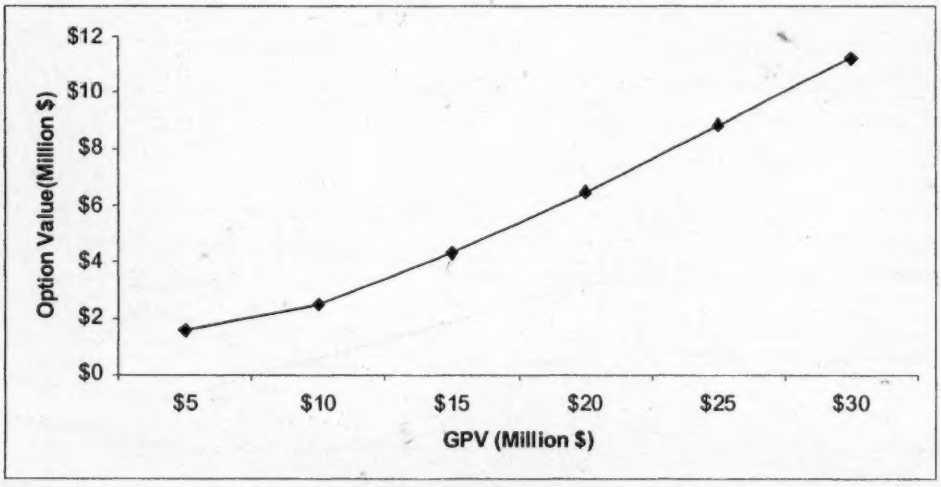

Figure 6.4: Sensitivity of the Multiple Option Value to Changes in Gross Present Value (GPV)

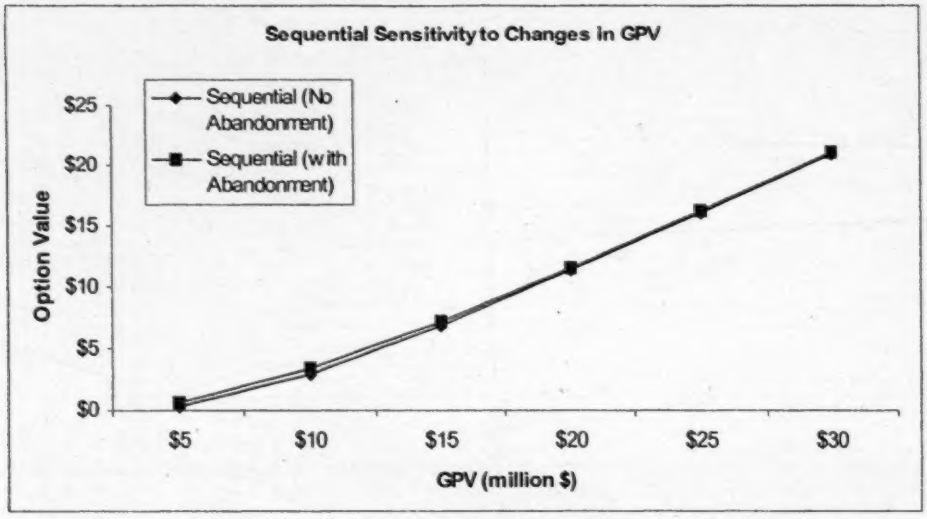

Figure 6.5: Sensitivity of Sequential Option Value to Changes in the GPV

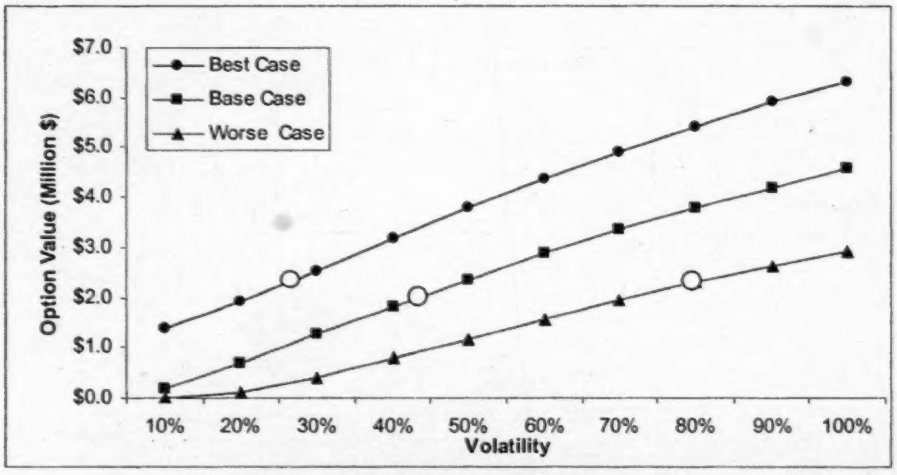

Figure 6.6: Sensitivity of the Option to Defer to Changes in the Volatility Under Different GPV Scenarios

**The circles in the graphs indicate option values where a change in volatility is associated with a change in the discount rate, with a resultant change in the GPV of the project.

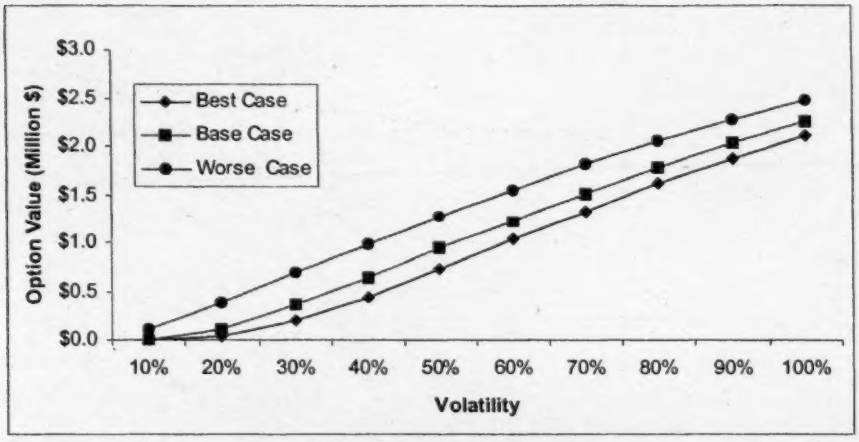

Figure 6.7: Sensitivity of the Option to Abandon to Changes in the Volatility Under Different GPV Scenarios

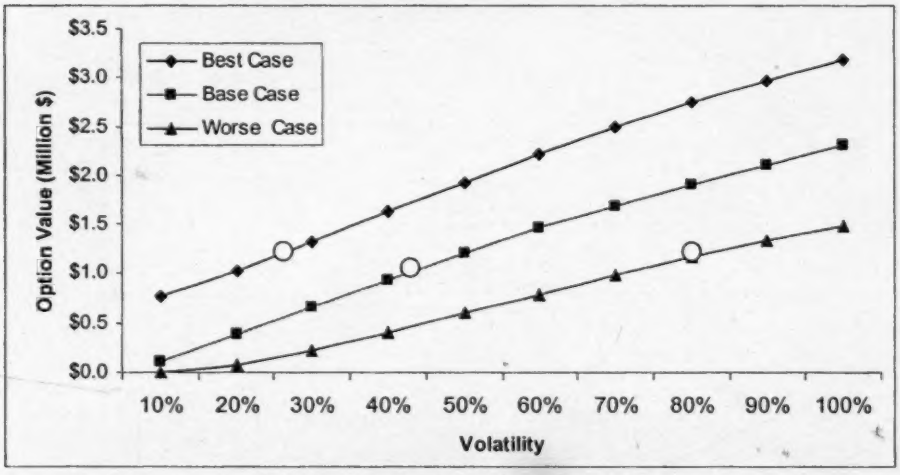

Figure 6.8: Sensitivity of the Option to Expand to Changes in the Volatility Under Different GPV Scenarios

**The circles in the graphs indicate option values where a change in volatility is associated with a change in the discount rate, with a resultant change in the GPV of the project.

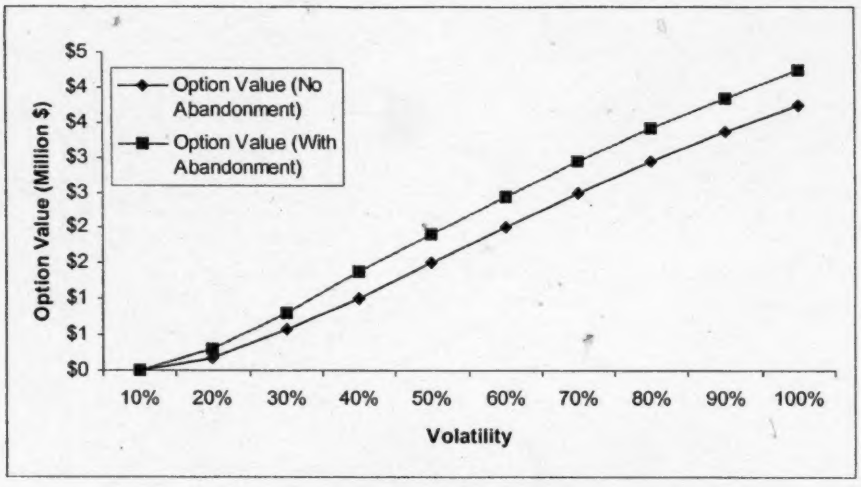

Figure 6.9: Sensitivity of the Sequential Option Values to Changes in the Volatility Under the Base Case Scenario

CHAPTER 7: SUMMARY AND CONCLUSIONS

7.1 NPV and Real Options Analyses

The main objective of this thesis was to determine the applicability of the real options approach to capital budgeting to investments in agriculture. As a result, both a static NPV and real option models were used to evaluate the capital budgeting investment decision of a 2600 farrow to finish hog project that has recently been set up in Western Canada. The static NPV gave a negative value for the hog project evaluated. This implies that the project is not worth undertaking and should be abandoned. However, this result contradicts the actual decision taken by the management of the hog project since they went ahead with the investment.

The hog project was then evaluated by considering the impact of strategic managerial decisions on the value of the project using real options valuation methods. Four different options were identified as being valuable for the hog project. These are the option to defer, abandon, expand and compound sequential option. The defer, expand and abandon options were evaluated with four different models and the results compared. In all the models the option to defer the beginning of the hog project was the most valuable. When this option alone is included in the valuation of the hog project, the project value becomes positive. The implication of this result is that the optimal decision for management of the hog project is to wait for better market conditions before undertaking the project. It also implies that the project has the potential of being profitable in the future and should therefore not be rejected.

When the hog project was evaluated with the separate values of the option to expand and the option to abandon, the decisions were similar to that of the static NPV. The value of the project was still negative in both cases implying it is not economically feasible for management to undertake the project if only one of the two options is present in the project. However, when the combined value of the options to abandon and expand was included in the valuation of the project the results indicated that the optimal investment strategy for management is to go ahead with the investment if there is no defer option. The results of the multiple options also indicated that options of opposite type with similar time to exercise are generally additive. When the ENPV of the project with the combined flexibility of the abandon and expand options was compared to the ENPV with the defer option, the results indicate that it would be optimal for management to wait for market situation to improve before undertaking the project when the exercise prices of the options are constant. However, when the exercise price of the options increase at the risk free rate over time, management can go ahead with the investment right now.

Comparing the real option results to the actual investment decision, the following reasons could possibly explain the actions of management to invest in the hog project. The first reason is that the estimated value of the option to defer the beginning of the project from this analysis may be higher than that perceived by the management of the project. This could be because the time to maturity for the defer option in this analysis was longer than what management anticipated. The second possible reason is that the option to defer might have diminished in value by the time the investment took place. The third possible reason is that management might have based their decision to invest in the project by considering the combined flexibility to expand or abandon the project or

anticipated increasing exercise price for the options over time. Each of the above three reasons could possibly influence management decision against deferring or rejecting the project.

Another important finding of this research is the impact dividend payments and increasing exercise prices had on the real option values of the hog project and the decision of management. Dividends and increasing exercise prices reduced the values of the option to defer, expand and the compound sequential option. This relationship is due to the fact that these options were valued analogous to American call options on the value of the project. On the other hand, dividend payments and increasing exercise prices enhanced the value of the option to abandon, which was valued as an American put option. If the impact of dividend is looked at in terms of loss of project value due to the actions of competitors, it could be inferred that competition decreases investors ability to defer and expand a project but enhance the value associated with abandoning an ongoing project. Since hog investments are carried out in a competitive environment in Canada, failure to account for its effects in the form of dividend payments on real option values could lead to under or over estimation of the option values depending on the type of option being considered. Similarly, if the exercise price of an option is not expected to be constant and this is not included in the valuation of the options, this may lead to estimation of incorrect option values.

The estimation of the sequential option implies that it would not be optimal for management to pay more than between 20%-25% of the initial gross project value as the cost of feasibility studies. In other words, the project is not worth undertaking if the cost of feasibility studies is higher than the estimated value of the sequential option. This analysis could not be compared to what management actually paid for feasibility studies since the information provided by the case study did not explicitly indicate how much was paid for the feasibility studies of the project.

The results of this study show that the inclusion of managerial flexibility can affect project value and lead to different conclusions regarding the profitability of investment in hog production. The results from the real option analysis may provide a better explanation of the actual decision taken by the management of the hog project relative to the static NPV model results. With the static NPV alone, the project should have been rejected but the real option analysis suggested otherwise. This result has demonstrated that recognising the options inherent in a hog investment opportunity and quantifying the values of such options may have a significant impact on the investment value and hence capital budgeting decision-making. The important result demonstrated from the implementation of the real options approach to investments in hog production is that options can be recognised as inherent in such projects and could possibly add value. Coming up with the appropriate value for the options may assist in properly assessing a project's value. The value of the real options in this project was large enough to over turn the decision implied by the static NPV analysis.

The option analysis carried out in this thesis did not use any complex mathematics other than the same algebra use for the NPV analysis. This makes it relatively easier for interested managers or investors who are already familiar with the NPV approach to also apply the real options methodology without much difficulty. The option theory approach holds promise for use not only in hog investment but investment in agriculture in general.

7.2 Sensitivity Analysis

The sensitivity analysis indicated that changes in the prices of hog and feed could play an important role in management decision to either invest now or defer the beginning of the project. For management to invest now, the expected average price of hogs has to increase by between 23%-29% else the project should be deferred, all other things being equal. The change in expected average hog price at which the project should start now reduced significantly when the combined flexibility to expand and abandon is considered. The results indicate that a long run average hog price increase of just between 4%-6% is enough for management to start the project now when considering the joint abandon/expand option. If the price of feed were to decrease from $74 to between $31-$41 per finished animal, it would be optimal for management to invest now. Similarly, when the project value is considered with the combined flexibility to expand and abandon a decrease of feed price by between 8% -13% should be enough for management to invest now, all things being equal. The average expected prices of hog determined by this analysis are not consistent with the history of the behaviour of hog price in Alberta. The analysis indicated that if the investment is almost completely reversible, there would be no need for management to defer the beginning of the project. This is because the project could be abandoned at any time and the initial investment recovered if market conditions deteriorate. A small decrease in average expected hog price or increase in feed price would be enough for management to reject the project. This is as a result of the fact that the NPV of the project is already negative and therefore a small unfavourable change in prices would lead to the project being abandoned.

Both the NPV and the gross project value decrease with an increase in the risk-adjusted discount rate of the project. However, there is a positive relationship between the volatility of the project value and the discount rate. The discount rate and the volatility may not be independent. If the volatility is being changed then it may be necessary for the discount rate to be changed simultaneously in order to obtain meaningful option value results. This is consistent with the theory behind the CML. The CML model shows the expected returns of a project as an increasing function of volatility of the project.

In the case of the impact of the gross project value on the values of the estimated options for the hog project, the results were as expected from the theory of option pricing. The options to defer, abandon and the compound sequential were an increasing function of the gross project value. This is because they were all valued analogous to American call whose value goes up as the project value increases. The option to abandon however was a decreasing function of the gross present value. This is implied by the fact that the option to abandon is valued as an American put option and put options decreases as the project value increases. The implication of this result is that as the project value increases, the expected benefits of the project also increases thereby decreasing the chances of abandoning the project and vice versa. This also increases the chances of expanding the project within the maturity date of the option.

The sensitivity of all the option values to changes in the volatility showed a positive relationship for all the scenarios considered. This is because as the volatility increases both the upside and downside risks of the project also increase thereby increasing the chances of the option being in the money. Option values were quite sensitive to changes in the volatility as expected from option pricing theory. However,

this sensitivity analysis may over estimate the impact of volatility on the option values. The analysis is not capturing the indirect impact, which results from an increase in the static NPV discount rate that may be associated with an increase in the volatility. An increase in the discount rate lowers the gross project value and therefore the call option values contingent on the project value. Thus, the total impact of an increase in volatility on call option values (e.g., defer, expand, etc.) may be indeterminate.

7.3 Study Limitations and Further Research

7.3.1 NPV

The first limitation of the NPV analysis of the hog project relates to the risk-adjusted discount rate used for the calculation of the NPV. The analysis determines the discount rate for the static NPV based on the main sources of risk associated with hog production. The discount rate determination was based on the assumption that hog investments fall under medium type risk investments. As a result, the rate may not reflect the exact level of risk associated with this hog project. Secondly, determining the appropriate initial investment cost was not straightforward to accomplish when investments are phased in over several time periods and production starts before building is completed, as was the case in this case study. The problem relates to the discount rate (i.e., risk free or risk adjusted) that should be used to discount future investments. This analysis assumed that future investments were uncertain and as a result discounted it with the risk-adjusted discount rate. Thirdly, the NPV analysis did not take into consideration the implications of income tax policies on the results of the analysis. Both federal and provincial income tax policies could have an impact on the results of the analysis and hence the investment decision of investors.

7.3.2 Real Options Analysis

In the real option analysis, models 2 and 4 assume that the exercise prices of the options increases at the risk free rate. This assumption was made for simplicity and may not be the appropriate approach to incorporating variable exercise prices in option valuation. Luehrman (1998) indicates that the best approach is to estimate the probability distribution of the exercise prices and use the uncertainty of the distribution to estimate the change in the exercise price over time.

In estimating the volatility of the project value, the prices were predicted based on the assumption of mean reversion stochastic process. Although the SUR results indicated some evidence of mean reversion in the historical prices used, there are several other stochastic processes (e.g., geometric Brownian motion etc.) that can also be modeled. In addition, the AR (1) process was used to predict the prices of hog and feed. There are several complex models that could be used to predict the prices such as including more lags, vector auto-regression models, and cointegerated models. These models might be more appropriate for forecasting prices.

Choosing the time to maturity variable for the real option analysis was not straightforward. There was no specific guide based on which the maturity date of the options could be determined. As a result, the time to maturity variables used for the option analysis was a subjective estimation and may not be the optimal time to maturity.

7.3.3 Sensitivity Analysis

Most of the sensitivity analysis conducted in this thesis is based on changing one variable at a time and examining its impact on the NPV and real options results, holding all other variables constant. This approach was used for simplicity but it may not be the case for most real life projects. Not only does this approach have the tendency of failing to account for the full impact of the variable being examined but also ignores the impact of a combination of variables at the same time, as it happens with most real life projects. For instance, the SUR results indicated a positive interdependency between the prices of hog and barley implying that an increase in the price of one should affect the price of the other. However, the sensitivity analysis overlooked this interdependency and as a result may not properly capture the full impact of the variables being changed. Related to this is the fact that changing the volatility in the real options model does not account for the indirect effects of the volatility on the options values through the discount rate on the GPV. As a result, the analysis may not properly account for the full impact of the volatility on the option values.

Further studies can be conducted to empirically estimate the discount rate for the NPV analysis using CML model. The sensitivity analysis showed that the NPV analysis is sensitive to the discount rate and therefore would be beneficial if a closer attention is paid to its estimation. Also the NPV analysis could be extended to account for the impact of both federal and provincial income tax policies on the results of the analysis.

Looking further into the estimation of the volatility of the value of the project could be beneficial to the analysis. For instance, this analysis used a mean reversion stochastic process and AR(1) model to predict the prices from the Monte Carlo simulation. Further studies may look at the impact of other stochastic process such as geometric Brownian motion or a different model for predicting the price process on the estimated volatility and for that matter the values of the options for the project.

One of the main purposes of this study was to quantify the value of the options identified for the hog project in combination. However, the problem of dependence of the option to wait and the other options remains to be appropriately modeled. The literature on option valuation does not give much information on how to model problems of this nature. Although some authors have valued, in combination, options that are not independent but they show little in terms of their valuation procedures. Further research is warranted in the valuation of dependent type options in combination.

The sensitivity analysis can be extended to determine the indirect impact of the volatility on the option values, particularly those valued analogous to call options. Finally, the findings of this study have demonstrated that the real options methodology can be applied to investments in agriculture. Similar research can be extended to investments in the other agricultural industries.

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APPENDIX A

Deriving the Binomial Model

In deriving the binomial option pricing formula, Cox, Ross, and Rubinstein (1979) assume that the underlying asset price (e.g. stock price) follows a multiplicative binomial process over discrete periods. The rate of return on the asset over each period can have two possible values, up or down with associated probabilities. If the current asset price is S, it can either rise to S_u or fall to S_d at the next period. These up and down movements have the corresponding probabilities of q and $(1-q)$ respectively. This is shown in figure 3.2 below.

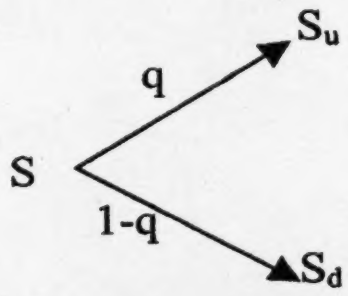

Figure A.1: One Period Stock Price Movement
Source: Cox et al., 1979

The model assumes that the interest rate is constant, and individuals may borrow or lend as much as they wish at this rate. In addition, it assumes no taxes, no transaction costs, or margin requirements. Hence, individuals can sell short any security and receive full use of the proceeds.

Figure 3.3 shows the value of a call option, C, on the asset, where C_u and C_d are the values of the option when the stock price is S_u and S_d respectively, and E is the exercise price.

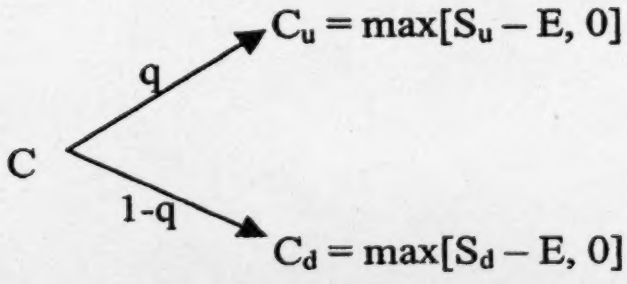

Figure A.2: One Period Movement of Call Option Value
Source: Cox et al., 1979

The value of put option, P, when the stock price is S_u and S_d are given as:

$$P_u = max[E - S_u, 0] \quad and \quad P_d = max[E - S_d, 0]$$

One of the basic ideas behind the pricing of options is that one can create an equivalent portfolio by buying a particular number of shares of the underlying asset (e.g., common stocks) and borrowing against them an appropriate amount at the risk free rate that would exactly replicate the future returns of the options in any state of nature. Suppose a portfolio is formed containing Δ share of stock and the dollar amount B in risk-free bonds.[37] The cost of the portfolio will be $\Delta S + B$, and the values of the portfolio at the end of the period for the up and down states respectively are:

$$\Delta S_u + (1+r)B$$
$$\Delta S_d + (1+r)B$$

Since the option and the equivalent portfolio would provide the same future returns, to avoid risk free arbitrage profit opportunities they must sell for the same current price. Thus the portfolio is constructed in such a way that it approximately replicates the option payoff at end of the period, whether the stock moves up or down. This requires that:

$$\Delta S_u + (1+r)B = C_u \qquad \text{(A.1)}$$

$$\Delta S_d + (1+r)B = C_d \qquad \text{(A.2)}$$

Solving these equations for Δ and B results in the following:

$$\Delta = \frac{C_u - C_d}{S(u-d)}, \qquad B = \frac{uC_d - dC_u}{(u-d)(1+r)} \qquad \text{(A.3)}$$

Having these ratios for the stock and the loan it is now possible to calculate the value of the option at time t. If there are to be no risk free arbitrage opportunities, the current value of the call option, C, cannot be less than the current value of the replicating portfolio, $\Delta S + B$. Thus the following must be true if there are to be no risk free arbitrage opportunities:

$$C = \Delta S + B \qquad \text{(A.4)}$$

Replacing Δ and B in equation A.4 results in the following value of the call option,

$$C = \frac{C_u - C_d}{(u-d)} + \frac{uC_d - dC_u}{(u-d)(1+r)} \qquad \text{(A.5)}$$

which simplifies to

$$C = \left[\left(\frac{(1+r)-d}{u-d}\right)C_u + \left(\frac{u-(1+r)}{u-d}\right)C_d\right] \Big/ (1+r) \qquad \text{(A.6)}$$

Equation A.6 can further be simplified by defining

$$p \equiv \frac{(1+r)-d}{u-d} \quad \text{and} \quad 1-p \equiv \frac{u-(1+r)}{u-d}$$

which represents the risk neutral probabilities that depend on the magnitude of the up and down movements and risk-free interest rate, and inserting this probability into equation A.6 results in the following equation for the value of a call option and a put option over one period;

$$C = \frac{pC_u + (1-p)C_d}{(1+r)}, \qquad P = \frac{pP_u + (1-p)P_u}{(1+r)} \qquad \text{(A.7)}$$

[37] Note that, buying and selling of bonds is the same as lending and borrowing respectively.

As can be seen from equation A.7 above, the value of the option in each node in the binomial tree is equal to the expected value of the call option in the next period discounted at the risk-free interest rate.

The formula (equation A.7) has a number of notable key features (Cox et al., 1979). First, the actual probability, q, does not appear in the formula for the option. The reason is that the actual probability distribution is already incorporated into the stock price and hence already in the option formula. This means that the model is independent of investors that have different subjective probabilities about an upward and/or downward movement in the stock. Second, the value of the call or put does not depend on investors' attitude towards risk since it was assumed that individuals prefers more wealth to less and therefore has the incentive to take advantage of profitable risk-free arbitrage opportunities. Finally, the only random variable on which the call or put value depends on is the stock price itself. In particular, it does not depend on the random prices of other securities or portfolios.

The formula for pricing a call when there is more than one period to the expiration date is a simple extension of the one-period formula just derived. Figure A.4 shows the binomial tree for two periods for the value of stock.

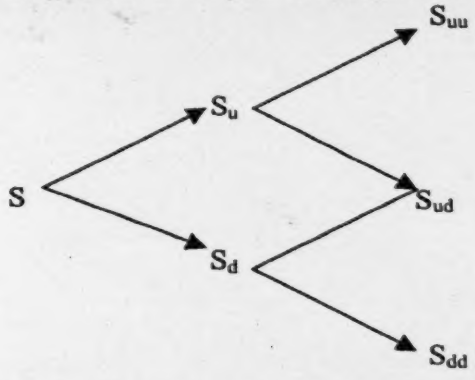

Figure A.3: A Two-Step Binomial Tree for Stock Value

Similarly, Figure A.5 shows the tree for the value of a call option for two periods.

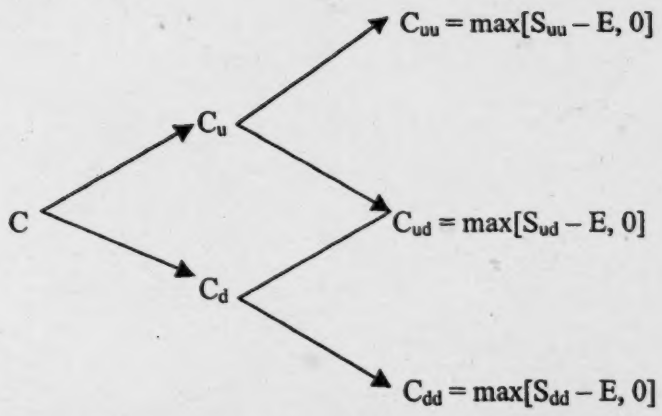

Figure A.4: A Two-Step Binomial Tree for Call Option Value

As before the subscripts show the value of the stock or call option at the different states, for example C_{uu} stands for the value of the call option after two consecutive upward moves. In order to value the call option at time t the value of C_u and C_d must be calculated at time $t+1$. These values in turn depend upon the value of C_{uu}, C_{ud}, and C_{dd} at time $t+2$. The values of C_u and C_d are given as[38]:

$$C_u = \frac{pC_{uu} + (1-p)C_{ud}}{(1+r)} \quad \text{and} \quad C_d = \frac{pC_{du} + (1-p)C_{dd}}{(1+r)} \qquad \text{(A.8)}$$

The procedure is to start at the end of the tree at time $t+2$ and calculate the value of the option at the previous time period $t+1$ and end up with a final value of the call option at time t.

Extending the binomial model into more periods will eventually make the calculations rather large so a general formula for the calculations is needed. The general binomial option pricing formula is shown in equation 3.14, where the variable a stand for the minimum number of upward moves that the stock must make over the next n periods for the call option to finish in-the-money. If $a > n$ the call option value would be zero because the option would never end up in the money (Cox et al., 1979).

$$C = S\Phi[a; n, p'] - Er^{-n}\Phi[a; n, p], \qquad \text{(A.9)}$$

where

$$p' = \frac{u}{d}p$$

C = call option value
a = the minimum number of upward moves that the stock must make over the next n periods in order for the call to finish in the money.
p = risk neutral probability
n = the number of periods until maturity
S = current stock price
E = exercise price

[38] Note that the presentation in equation A.8 is a discrete approximation. The equation becomes a continuous approximation when the (1+r) is replaced with the exponential of the risk free rate (e^r).

r = risk-free rate over one period
$\Phi[\cdot]$ = complementary binomial distribution.

Two of the important inputs in the binomial formula are the values of u and d. These values are based on the standard deviation of returns (volatility) of the stock (S) and the number of n intervals until expiration over the time period t. The formulas are:

$$u = e^{\sigma\sqrt{\Delta t}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\Delta t}} = 1/u$$

The values of u and d represent the continuously compounded rates of returns if the stock moves up or down respectively. These values of u and d are required in every period in order for the n periods to have a binomial distribution with the given standard deviation. Cox et al. (1979) prove that this method corresponds to the Black-Scholes model as $n \to \infty$.

APPENDIX B

Mean Reverting Stochastic Process

One common stochastic processes used to model prices is the mean reversion stochastic process (Zeng, 2001). Mean reversion is a property of a stochastic process where the variable value tends to revert back to some normal level when it is away from that level. This normal level is often called the mean or trend of the variable underlying the stochastic process. The concept of mean reversion characteristics of real commodity prices is backed by the economic notion that in equilibrium, the expectation is that when prices are relatively high, supply will increase since higher cost producers of the commodity will enter the market thereby putting a downward pressure on prices, all things being equal. Conversely, when prices are relatively low, supply will decrease since some of the higher cost producers will exit the market, putting upward pressure on prices. This impact of relative prices on supply of the commodity will induce mean reversion in commodity prices.

The simplest and most well known mean reverting process is the Ornstein-Uhlenbeck process, popularized by Vasicek (1977) as a model of how interest rates behave, is given as[39]:

$$dP_t = \eta(\overline{P} - P_t)dt + \sigma dz_t \qquad \text{(B.1)}$$

where,

P_t = spot price at time t

η = rate or speed of mean reversion

$\overline{P}$ = the long term equilibrium value of P_t

dt = change in time

σ = volatility of the price process

dz = an increment of a Weiner process.

If $\overline{P} < P_t$, the process exhibits a negative drift and when $\overline{P} > P_t$ the drift is positive (assuming $\eta > 0$). Hence, the real price of commodities is driven back to its long-term average, $\overline{P}$, with a speed of reversion η. One of the characteristics of the Ornstein-Uhlenbeck process is that the change in price is normally distributed. If a commodity price follows equation A5.1 and the current price is P_0, then its expected value at time t is given as (Dixit and Pindyck, 1994):

$$E[P_t] = \overline{P} + (P_0 - \overline{P})e^{-\eta t} \qquad \text{(B.2)}$$

and the variance $(P_t - \overline{P})$ is:

$$V(P_t - \overline{P}) = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta t}) \qquad \text{(B.3)}$$

The process in equation A5.1 becomes a random walk as $\eta \to 0$.

In contrast to a random-walk specification, the mean-reversion process is less volatile because it keeps pulling back commodity prices toward the mean level. However, the price from a mean-reverting process also has a smaller chance of reaching higher

[39] Dixit and Pindyck (1994) cover this discussion in more detail.

levels than the price from a random walk, implying smaller returns (Zeng, 2001). The Ornstein-Uhlenbeck mean reversion process in equation A5.1 is also criticized for its lack of economic appeal, in that negative prices are possible.

Based on the last criticism, Cox et al. (1985) modified equation A5.1 to ensure non-negativity of prices[40]. The model, known as the square root process, is given as[41]:

$$dP_t = \eta(\overline{P} - P_t)dt + \sigma\sqrt{P_t}dz_t \qquad \text{(B.4)}$$

The variables in this model are the same as previously defined in equation A5.1. According to Cox et al. (1985), the price behaviour implied by equation A5.4 has the following properties: 1) due to the presence of the square root, negative prices are precluded, and that if the price reaches zero, it can subsequently become positive, 2) the absolute variance of price increase when price itself increases, and 3) there is a steady state distribution for the price.

Schwartz (1997) and Dixit and Pindyck (1994) show that the mean reversion process in equation B.1 and B.4 are similar to the AR(1) model used in Monte Carlo simulation analysis in chapter 5[42]. However the model in chapter 5 assumes an arithmetic price process. Schwartz models the AR(1) process using the log of prices.

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[40] They used interest rate in their derivation.

[41] See Cox, Ingersoll and Ross (1985) for detailed derivation of the model.

[42] Schwartz (1997) and Dixit and Pindyck (1994) show the derivation of the AR(1) model from the mean reversion stochastic process. Schwartz shows that $dP_t = \eta(\overline{P} - \ln P_t)P_t dt + \sigma P_t dz_t$ is equivalent to B.1 when prices in B.1 are in log prices. The appropriate AR(1) model for this process is $\ln P_t = \alpha + \beta \ln P_{t-1} + \varepsilon_t$.

APPENDIX C

This appendix demonstrates how the binomial option-pricing model is used to determine the values of the real option in the hog project. In this example, only the valuation of the option to defer the hog project using the continuous dividends and constant exercise price approach (Model 1) is demonstrated. The evaluation of the other options follows the same process. Figure C1 below depicts the forward calculation of the value of the hog project in the binomial tree with no dividend adjustment. The binomial tree in Figure C1 represents all the possible values of the hog project during the life of the options. In this analysis, the maturity date of the options for the hog project was set at 5 years with a time interval *(Δt)* of 0.2. This implies that the value of the hog project would move 25 times during the life of the options. Figure C1 starts with the initial gross present value of the project ($7.1M) at time period $t = 0$. At the next step of the tree *($t=1$)*, the value of the hog project could either move up or down. The up and down ratios for this project were calculated to be 1.21 and 0.83 respectively[43]. Thus at time period $t = 1$, the value of the project could move either up to $8.59M (i.e., the initial value of $7.1M multiplied by the up ratio of 1.21) or down to $5.89M (i.e., $7.1M multiplied by the down ratio of 0.83). In time period $t = 2$, the values of the projects are determined by following the same procedure as $t = 1$. The value in the uppermost node in this time period, $10.48M, is calculated by multiplying the up value of the pervious period *($t = 1$)* by the up movement value ($8.59M times 1.21)[44]. The value of the project in the middle note at time period $t = 2$ by using either the value at the uppermost or the down node in the previous period. Using the uppermost node in time period $t = 1$, the value of the project in the middle node at period $t = 2$, $7.1M, is calculated by multiplying the down movement value by the uppermost value (i.e., $8.59M times 0.83). In calculating the value ($4.81M) at the down node in $t = 2$, the value at the down node in $t = 1$ is multiplied by the down ratio (i.e., $5.84M times 0.83). The values of the hog project for the remainder of the binomial tree are calculated by repeating the process until the value of the project at the end nodes of the tree are determined at $t = 25$[45]. The values at the end nodes of the tree are the possible values of the project at the maturity of the option. Once all the possible values of the hog project have been determined in the binomial tree, the value of the option can be calculated by working backwards from the end nodes of the tree to present (t= 0).

Figure C2 presents the backward calculation of the value of the option to defer the beginning the hog project. The defer option is valued similar to American call option, with the exercise price equal to the initial cost of the investment. The initial capital cost of the hog project was calculated to be $8.59M. The value of this option is determined by first calculating the intrinsic values of the option at the end of the binomial tree *($t = 25$)*. If the intrinsic value at any of the end nodes is greater that zero, then management will invest if the value of the hog project ends up in that node. However, management will not invest in any end node that has an intrinsic value of zero.

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[43] See section 5.4.3 for the calculation of the up and down ratios.

[44] Note that the value may not be exactly the same as those on the binomial tree due to rounding.

[45] For further discussion on valuing real options using the binomial tree, interested readers are referred to Copeland and Antikarov (2001).

Starting at the uppermost end note at time period $t = 25$ in Figure C1 which has a value of \$919.24M, the intrinsic value of the option to defer the hog project in the uppermost end node in Figure C2 is calculated using equation 4.1 as follows:

$$\text{Max } [\$919.24M - \$8.59M, 0] = \$910.65M$$

Thus, the intrinsic value of the option to defer at the uppermost end node of the binomial tree is \$910.65M, implying that management will invest if the value of the project ends up in that node. The same equation is used to calculate the remainder of the intrinsic value of this option in Figure C2. For instance, the intrinsic value at the bottom end node of the tree is calculated using the value at the bottom end node of Figure C1 as follows:

$$\text{Max } [\$0.05M - \$8.59M, 0] = 0$$

This implies that management will not invest if the value of the hog project ends up in this node within the maturity date of the option. Once all the values at the end nodes are calculated, the next step is to move back to time period $t = 24$ to determine whether the option should be exercised or kept alive. This is done using equation 3.15 in chapter 3. Using the uppermost node in Figure C1 at $t = 24$, the value of the option if exercised in Figure C2 is calculated as follows:

$$\$756.73M - \$8.59M = \$748.14M$$

On the other hand, the value of the option if kept alive is calculated using the intrinsic values of the uppermost and second uppermost nodes in time period $t = 25$ in Figure C2. This is calculated as follows:

$$= e^{(-0.04*0.2)}[0.45*\$910.65M + 0.55*\$614.36M] = \$741.73M$$

The decision at this node is made by comparing the two values and choosing whichever is larger as follows:

$$\text{Max } [\$748.14M, \$741.73M] = \$748.14M.$$

From the above calculations, the value of the option if exercised is greater than the value of the option if kept alive at this node. The optimal decision for management therefore would be to invest in the hog project if its value ends up in this node within the next five years. The values of the options for the rest of the nodes are determined using the same procedure and comparing the values. The process continues working backwards for the rest of the nodes until the value of the option to defer is obtained at time period $t = 0$. In this example, the value of the option to defer the hog project is \$1.79M.

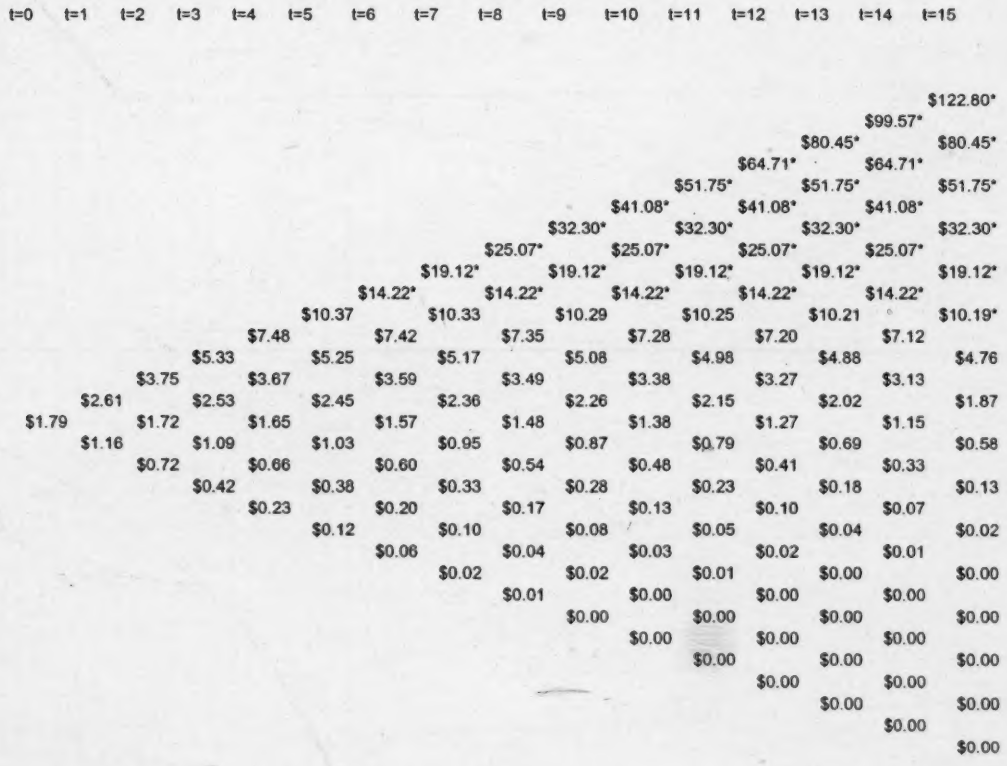
In Figure C2, all the values in the binomial tree with asterix (*) indicate the nodes in which management will exercise the option to defer the hog project (i.e., invest in the hog project) if the value of the project ends up in any of the nodes. In these nodes, the value of the option if exercise is greater than the value of the option if kept alive. All the other values in the tree without asterix indicate the nodes in which management will keep the option alive if the value of the project ends up in those nodes. The nodes with zero values imply that the option has no value and therefore the option would neither be kept alive nor exercised if the value of the project ends in any of those nodes.

Figure C1: Forward Calculation of the 2600 Hog Project's Value in the Binomial Tree

t=0	T=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10	t=11	t=12	t=13	t=14	t=15
															131.39
														108.16	
													89.04		89.04
												73.30		73.30	
											60.34		60.34		60.34
										49.67		49.67		49.67	
									40.89		40.89		40.89		40.89
								33.66		33.66		33.66		33.66	
							27.71		27.71		27.71		27.71		27.71
						22.81		22.81		22.81		22.81		22.81	
					18.78		18.78		18.78		18.78		18.78		18.78
				15.46		15.46		15.46		15.46		15.46		15.46	
			12.73		12.73		12.73		12.73		12.73		12.73		12.73
		10.48		10.48		10.48		10.48		10.48		10.48		10.48	
	8.59		8.62		8.62		8.62		8.62		8.62		8.62		8.62
7.10		7.10		7.10		7.10		7.10		7.10		7.10		7.10	
	5.89		5.84		5.84		5.84		5.84		5.84		5.84		5.84
		4.81		4.81		4.81		4.81		4.81		4.81		4.81	
			3.96		3.96		3.96		3.96		3.96		3.96		3.96
				3.26		3.26		3.26		3.26		3.26		3.26	
					2.68		2.68		2.68		2.68		2.68		2.68
						2.21		2.21		2.21		2.21		2.21	
							1.82		1.82		1.82		1.82		1.82
								1.50		1.50		1.50		1.50	
									1.23		1.23		1.23		1.23
										1.01		1.01		1.01	
											0.84		0.84		0.84
												0.69		0.69	
													0.57		0.57
														0.47	
															0.38

t=16	t=17	t=18	t=19	t=20	t=21	t=22	t=23	t=24	t=25
									919.24
								756.73	
							622.95		622.95
						512.82		512.82	
					422.16		422.16		422.16
				347.53		347.53		347.53	
			286.09		286.09		286.09		286.09
		235.52		235.52		235.52		235.52	
	193.88		193.88		193.88		193.88		193.88
159.60		159.60		159.60		159.60		159.60	
	131.39		131.39		131.39		131.39		131.39
108.16		108.16		108.16		108.16		108.16	
	89.04		89.04		89.04		89.04		89.04
73.30		73.30		73.30		73.30		73.30	
	60.34		60.34		60.34		60.34		60.34
49.67		49.67		49.67		49.67		49.67	
	40.89		40.89		40.89		40.89		40.89
33.66		33.66		33.66		33.66		33.66	
	27.71		27.71		27.71		27.71		27.71
22.81		22.81		22.81		22.81		22.81	
	18.78		18.78		18.78		18.78		18.78
15.46		15.46		15.46		15.46		15.46	
	12.73		12.73		12.73		12.73		12.73
10.48		10.48		10.48		10.48		10.48	
	8.62		8.62		8.62		8.62		8.62
7.10		7.10		7.10		7.10		7.10	
	5.84		5.84		5.84		5.84		5.84
4.81		4.81		4.81		4.81		4.81	
	3.96		3.96		3.96		3.96		3.96
3.26		3.26		3.26		3.26		3.26	
	2.68		2.68		2.68		2.68		2.68
2.21		2.21		2.21		2.21		2.21	
	1.82		1.82		1.82		1.82		1.82
1.50		1.50		1.50		1.50		1.50	
	1.23		1.23		1.23		1.23		1.23
1.01		1.01		1.01		1.01		1.01	
	0.84		0.84		0.84		0.84		0.84
0.69		0.69		0.69		0.69		0.69	
	0.57		0.57		0.57		0.57		0.57
0.47		0.47		0.47		0.47		0.47	
	0.38		0.38		0.38		0.38		0.38
0.32		0.32		0.32		0.32		0.32	
	0.26		0.26		0.26		0.26		0.26
		0.21		0.21		0.21		0.21	
			0.18		0.18		0.18		0.18
				0.15		0.15		0.15	
					0.12		0.12		0.12
						0.10		0.10	
							0.08		0.08
								0.07	
									0.05

Figure C2: Backward Calculation of the Value of the Option to Defer for the 2600 Hog Project.



t=16	t=17	t=18	t=19	t=20	t=21	t=22	t=23	t=24	t=25
									$910.65*
								$748.14*	
							$614.36*		$614.36*
						$504.23*		$504.23*	
					$413.57*		$413.57*		$413.57*
				$338.94*		$338.94*		$338.94*	
			$277.50*		$277.50*		$277.50*		$277.50*
		$226.93*		$226.93*		$226.93*		$226.93*	
	$185.29*		$185.29*		$185.29*		$185.29*		$185.29*
$151.01*		$151.01*		$151.01*		$151.01*		$151.01*	
	$122.80*		$122.80*		$122.80*		$122.80*		$122.80*
$99.57*		$99.57*		$99.57*		$99.57*		$99.57*	
	$80.45*		$80.45*		$80.45*		$80.45*		$80.45*
$64.71*		$64.71*		$64.71*		$64.71*		$64.71*	
	$51.75*		$51.75*		$51.75*		$51.75*		$51.75*
$41.08*		$41.08*		$41.08*		$41.08*		$41.08*	
	$32.30*		$32.30*		$32.30*		$32.30*		$32.30*
$25.07*		$25.07*		$25.07*		$25.07*		$25.07*	
	$19.12*		$19.12*		$19.12*		$19.12*		$19.12*
$14.22*		$14.22*		$14.22*		$14.22*		$14.22*	
	$10.19*		$10.19*		$10.19*		$10.19*		$10.19*
$7.05		$6.96		$6.88		$6.87*		$6.87*	
	$4.62		$4.46		$4.30		$4.14*		$4.14*
$2.98		$2.79		$2.58		$2.30		$1.89*	
	$1.70		$1.49		$1.22		$0.84		$0.03*
$1.00		$0.84		$0.63		$0.38		$0.02	
	$0.46		$0.32		$0.17		$0.01		$0.00
$0.25		$0.16		$0.08		$0.00		$0.00	
	$0.08		$0.03		$0.00		$0.00		$0.00
$0.04		$0.02		$0.00		$0.00		$0.00	
	$0.01		$0.00		$0.00		$0.00		$0.00
$0.00		$0.00		$0.00		$0.00		$0.00	
	$0.00